



# Projet QCSP

WP1 : PN sequences

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V1.0

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**Abstract:** This document gives the PN sequences found by a matlab program that tries to find better solution than LFSR based PN sequences construction.

## History:

V1.0: January the 24, 2020, Proposition of PN sequences.

## 1 PROPOSITION OF NEW PN SEQUENCES.

In this document, we gives the PN sequence found by a matlab program.

Table I compares the new obtained sequences with the one proposed in D1.1.

PN Length	$L_{+\infty}$ -norm		$L_1$ -norm		$L_2$ -norm
	LFSR [1]	Proposed	LFSR [1]	Proposed	Proposed
64	12	8	2.48	2.28	11.17
128	20	16	4.94	4.375	34.75
256	24 or 26	16	6.95	5.50	46.37
512	32	32	10.07	7.34	87.87
1024	56	48	13.32	11.17	204.15
2048	76	76	19.85	17.61	494.72
4096	132	128	28.16	23.65	894.20

Table 1: Comparison of the proposed sequence with LFSR sequence.

Let  $q$  be the size of the PN sequence  $P_0$ . The auto-correlation vector is given as

$$\theta(k) = \sum_{i=0}^{q-1} P_0(i)P_0(i+k),$$

where addition are done modulo  $q$ .



We define the  $L_1$ ,  $L_2$  and  $L_{+\infty}$  norms of  $\theta$  respectively as:  $L_1(\theta) = \frac{1}{q} \sum_{k=1}^{q-1} |\theta(k)|$  ;  $L_2(\theta) = \frac{1}{q} \sum_{k=1}^{q-1} \theta(k)^2$  and  $L_{+\infty}(\theta) = \max(|\theta(k)|, k = 1, \dots, q - 1)$ .

## 2 PN SEQUENCES.

PN64 = [  
 1000111110100110100010000100101001110111001000100111110101000111  
 ];

P128 = [  
 11111111001110101111011101110111011101100000001110000001010010110101010111010  
 10010111000000000110010111011000001100101110000011000110  
 ];

PN256 = [  
 011110100010110000101011111111010000011110100100000011011111010010101110  
 0000110101000010101000101001101110111100010110011100100011011010101111110  
 0010001101001110100111011001111101001101101101011000000110001000011000001  
 0000101101100111100111001000111000011  
 ];

PN512 = [  
 10000010010001100010101010110010111101111111000001111001110111000110000  
 0111001011001010001001001010010111110101101111001000110000000000011010000  
 1111110100110100010101000011101100011000011111010000101011010111111000010  
 1000101010011010110110000000010010000011011010111000001000100000100101010  
 011011100110110001111111011100101100110011110101100000111111100111010011  
 1011111100111000110101010111010111101101100100001001110100100000101000111  
 100100010110111010111001010101100001110100110111010000111111101000011001  
 0  
 ];

PN1024 = [  
 1001000011100101110100101000101000100000001100101110011110011011101001100  
 11000111110111111101010101110101011100001001110110110101000110011011000  
 001110110000010010011111111011111000010010100110001111010111101111111001  
 0010111010011011000100101001110111010011000010101010000110110001000010101  
 111101011001110101111000000000000000000100100011000011001001000001111010  
 1000011001101111010001001001010101011001010011111001000110111001110011011  
 1001110011111111001000010100111001001010011000010011111100001001111010010  
 10000000011001011010110000111111101011101011100001010001111010000100110  
 110111000110111110010111001000111000001011000011111000010111101100000111  
 000101111111010000101100100001001111100111010011111000011101011011000101  
 0000111100110001000101100010001011001011110011100101000101001000000010110  
 1011011100111011101011001001000001101001001100101101010000111011100101001  
 ];



```

010000111111001000101111010011011100000011001011111110001001000101000111
1111111010010001001011100111101011111000001010010011011000100110001100000
11
];

```

```

PN2048 = [
111111001111110000000111001000101000100101100110111110011001011001101101
0001111110001100010100101100010011110101000010111000100111010110110100101
10100010111011010100011100000011101000100010011011101111110010010011110
0111011011100101100101011010111010000110111110000110100110010001001111001
01001110001001110101101100101101100001111010110010010110101111111000100
011000100110001001011111100110000110000100110000111100111001011110111111
0100111010001000101110110100101001000010110001100001111110110100100001111
1011111001011001111101110111011110001011010001011010011011110000111001011
1011010110111001000110101010111100000110100111111100011011000100101111010
100110101010111010001110111010000011101000101111111000010000101100001011
1110011001001010000111011101000100001101010010110101001100101110001000011
1010011000000010000110001100110101101100101101100010000101010111101010100
000001111110011010100010110100011101100010010101010001101011001010111111
0010111111000001101010010010111111101011001000010100011101010011101011110
011010010011100000110010110000000101111010011011111110010000100110100101
1100110000101000011100001111010111001110010010010110111110110000000010000
1000110000101101000100001010101001110011011000011101001000010110101000110
0011011101110110011100101011011111000000000000000111010010101000111001011
110111101000100100001110110001110001110111000000101100011010110011000000
0000000010100000111110101000001000100011110110000110110111100110110011101
001011001010111111101010011100000000111010111001001000101100111000010111
01001011000010101100011100011100010001011101110011111111111111110111110000
11101010011111100110100001011010001101100010010001100110000000000000111010
0000011011000100110111101010000111111011101000001111110100011110011100111
010101010101010100111010101010111111111111010111111011100000000100000000
0011011001010011101011110010010011111001111100000011111110111001010001101
0100000001100100100101010101000111000100000100011101110000011101101000010
0100011001101101110010000110100000010011001111011000100010110101000000110
1001
];

```

```

PN4096 = [
1010000001110101100110100110111001001111000110100011101111101001101000011
1111110011010000110000101101001101010010010100011000000000001100110111001
11010110000010001100001101011011001101001101110010011110011100001110011111
1000001011010100100110101010010011011110111000110011111001001111000100001
00100111010111000110110100011000001101110010110001101000010110010000001111
000000101011000101010110010111011111001010001101110100000010111110001001
1110000001010110000110101100110110100100100001101111000010101000011110100
0110000010100011010101000001010001010001010101101010100110010001101101011
1110001101011101011001111110101010011101100100001111110001110000000000001
0111000100101101111011100010100011001010101010100110100000100110011111100
1001010110011111111111110100100001101000100110101101110011011001000101000
1000111011101010110111010110100010000000000110000011100000100100010101111

```



```

110100011110110001100010001111101100011010000100000001111101101000100100
1010000010010000101111110010110011000010000001111010110100011101101101000
1100010110101110001010110111101011001010110101011010010100111011111001011
0000010100011011000100111011101010001011001010110000110100110011100011010
100101001010011000101110000000000001111110001100111000010010011100001101
00100011010001010110101001100100011101010110110101101011010110111101101
1000010001111111010000001010110011000000100100011001101001101011011101010
0000010111000000110100001111011000000011101101111011110001101010001101010
001000100110101100110110011101010110000101011110000111111011111011111110
0001010100101111100001011001010111110001001011110011111100010011010101010
000001101011000001111011111000011000100101010100010000011101010100011000101
1111011001011110110000000001000011010110000101111101110001011010011011011
0101101110101000100010110110101001100001001111010001001000000100101110100
0010110000110001111100110001000110000000101011110100111011000010111111110
0110110000100010110001011101110011110000101010010000010000011101000110011
1110111100110010010011100101110010110101001001001011111000111000111100101
110011101110100111001100110111110011110011111100010110101000110010111110
010110000100011101001101001011000101100110101000101100011111110010110100
000001101000000010100110100111011000010000011001001110011001111010101100
1111100110100010001010101011111000001000011011110011110000101010000011000
1011011110011110011111010110110001110100111100010110111001100110001010011
0010010101000010001110110001000110111101011110111111000000111101000000100
0110100111011011101000101100111111111111010111000110100101110100111001111
0010110000101001011011110010001110000101101111110001001011010001001110010
1100110111010001111010001010011011000111000101110100011011011011111010100
1011000101110001111111100100011001011001000110110001011001100100101100100
1011000110001000110001011101010011111011000001001010111100000111100010001
1000000101110101000010111110110000010010111000110100110000000111001001010
1011110101010100010111010001110100101111100010101100011001011001101110100
1111001010001000110100001010110001101100010100100001011100001001010001101
0111010111011001110110110000111011100111010000111111101101100001010101110
1001011111011101110110011101100001111110011000000111111110101100000101000
00100000011100010011111111011111111111111111111111111111111011111110111111010
1001100100001110110101000111111110000001111111111001010100100010000110100
0101101000110011110101101110111111100100100110011101000011000111101000000
0110010000011111111111110000101110000100011110001101011010011100111011011
1000001010010001100011100011001111011010101111111110010100010001100000000
0111111111101001111101000011011100010100000011011001101101011001010101110
0010111111100101101110001111010110111001000100000001000101111101101001001
10100111100000100000001000100000111111111111111101010110000111111011000100100
1010010100001011101011000110110000010000111010100111011111100010000100110
111001001011010011000000000100001111110001010110000101111111000000001100
1101010110100100110000110001010000001111101010010100011110011010101000000
0010111001110011000110011111111111111110110100001011110000100110110010011011
10010100
];

```

### 3 MATLAB CODES.

The matlab code used to generate the PN sequence is given here after.

```
%=====
% E. Boutillon
% 16/12/2019
% Recherche de bonnes séquences PN selon le critère L2.
%=====
clear all

p = 12;
q = 2^p

for repetition= 1:50

%Choix au hasard d'une polution de 200 séquences.
pn = sign(randn(200,q));

for l = 1:40000 % Nombre de génération.
    for k = 201:400 % On double la population par croisement des 200 séquences
        initiales.
            u = ceil( rand(1,1)*255);
            a = floor(rand(1,2)*99)+1;
            if a(1) == a(2)
                a(2) = mod( a(1) + 1, 99) + 1;
            end
            pn(k,:) = [pn(a(1), 1:u) pn(a(2), u+1:end)];
            % On équilibre la séquence pour que le nombre de 0 soit égal au nombre
            % de 1.
            A = ( pn(k,:) == 1);
            m = sum(A) - q/2;
            if m < 0
                B = find( pn(k,:) == -1);
                for i=1:-m
                    pn(k, B(i)) = 1;
                end
            end
            if m > 0
                B = find( pn(k,:) == 1);
                for i=1:m
                    pn(k,B(i)) = -1;
                end
            end
            % Calcul de la fonction d'autocorrelation.
            U = ifft( fft(pn(k,:)).*conj(fft(pn(k,:))));
            score(k) = sum(U(2:end).^2)/q;
        end
        [v I] = sort(score);
        if mod(l, 20) == -1
```



```
fprintf('it n° %3d: ',1);
for i = 1:18
    fprintf('%2.2f ', score(I(10*(i-1)+1)));
end
fprintf('\n');
end
% On ne sélection que les 100 meilleurs candidats pour le round suivant.
pn = pn(I(1:100),:);
% On enrichi la population de 100 candidats au hasard.
pn(101:200,1:q/2) = sign(rand(100, q/2));

for i = 1:200
    pn(i,:) = [pn(i, i:end) pn(i, 1:i-1)];
    U = ifft( fft(pn(i,:)).*conj(fft(pn(i,:))));
    score(i) = sum(U(2:end).^2)/q;
end
end

U = ifft( fft(pn(1,:)).*conj(fft(pn(1,:))));
L2 = sum(abs(U(2:end)).^2)/q;
L1 = sum(abs(U(2:end)))/q;
Linf = max(abs(U(2:end)));
fid = fopen('result_q4096.txt','a');
fprintf(fid,'L1 norm : %2.3f, L2 norm : %2.3f, Linf_norm %2.3f\n',L1, L2,
Linf);
for i = 1:q
    fprintf(fid,'%1d',(pn(1,i)+1)/2);
end
fprintf(fid,'\n=====\n');
fclose(fid);
end
```