The Best, the Requested, and the Default Elementary Check Node for EMS NB-LDPC Decoder

Joseph Jabour$^{1,2}$, Cedric Marchand$^1$, and Emmanuel Boutillon$^1$

Université Bretagne Sud, Lorient, France$^1$
Lebanese International University, Beirut, Lebanon$^2$

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# Table of Contents

1. Non-Binary LDPC Codes and Decoders

2. Forward-Backward Extended Min-Sum Algorithm for NB-LDPC

3. The Best, Requested, and Default Algorithm
   - The Best, Requested, and Default Algorithm
   - Forward-Backward BRD Decoder

4. Complexity Analysis and Simulation Results

5. Conclusion

6. References
NB-LDPC codes are extension of binary LDPC codes with \( GF(q = 2^p) \) where \( p > 1 \) [1].

Information block of size \( K \) symbols on \( GF(q) \) is encoded to a code block of size \( N \) symbols by adding \( M \) redundant symbols.

Decoder consists of \( M \) Check Nodes (CNs) and \( N \) Variable Nodes (VNs).

Each CN \( C_i \) is connected to \( d_c \) VNs, denoted as check degree of connectivity.

Each VN \( V_j \) is connected to \( d_v \) CNs, denoted as variable degree of connectivity.
Non-Binary LDPC Codes and Decoders

Figure 1: Tanner Graph of NB-LDPC Decoder

$N = 9$

$M = 6$

$d_v = 2$

$d_c = 3$
Figure 2: An Edge Connecting VN $V_j$ to CN $C_i$. 
Complexity of NB-LDPC decoder is dominated in its CNs.

Extended Min-Sum (EMS) is proposed in [2] to reduce complexity of CNs.

How?

- By truncating size of messages from $q$ down to $n_m$.
- Reducing check node operations from $q^2$ down to $n_m^2$. 
Forward-Backward approach [3] implements EMS algorithm by decomposing CN into 3 layers each of $d_c - 2$ Elementary Check Nodes (ECNs).

**Figure 3:** CN Decomposition in Forward-Backward Approach for $d_c = 12$
Each ECN has only two inputs denoted as \((U^\oplus, U^+)\) and \((V^\oplus, V^+)\) each of size \(n_m\).

A matrix \(T_\Sigma\) is generated as

\[
T_\Sigma^+[u][v] = U^+[u] + V^+[v],
T_\Sigma^\oplus[u][v] = U^\oplus[u] \oplus V^\oplus[v]
\]

\(T_\Sigma^\oplus\) and \(T_\Sigma^+\) correspond to vector of GF symbols and their LLR values respectively.

ECN generates \(n_m\) candidates (GF and LLR couples) sorted in descending order of their reliability.
The Best, Requested, and Default (BRD) algorithm [4] is a generic NB-LDPC decoding algorithm.

- Allows VNs to request specific symbols from CNs.
- Requested symbols preserve decoding performance with shorter message sizes.
- Uses compression and decompression block at each side of edge.
Figure 4: Toy Example of BRD Decoder on GF(8) with $n_{VC} = 3$, $n_B = 2$ and $n_R = 2$
For BRD algorithm to be compatible with FB algorithm, a variant ECN is needed, called BRD-ECN.

BRD-ECN composed of two sub-blocks and three input vectors as shown

Figure 5: Structure of BRD-ECN
White ECN blocks $\rightarrow$ Conventional ECNs as in [3].

Grey ECN blocks $\rightarrow$ BRD-ECNs.

Figure 6: Forward-Backward BRD Architecture for $d_c = 12$
Integrating BRD algorithm with FB algorithm reduces complexity of CN and VN units by reducing
- Communication load.
- Sorters size.
- Arithmetic operations (real additions, GF additions, GF multiplications).
- Memory Allocations.
### Table 1: Size of Exchanged Messages per Edge on GF(64)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Code Rate</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$n_{vc}^{\oplus}$</td>
<td>$n_{vc}^{+}$</td>
</tr>
<tr>
<td>FB-BRD</td>
<td>$r \geq 5/6$</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$r = 1/2$</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$r = 1/3$</td>
<td>13</td>
<td>12</td>
</tr>
</tbody>
</table>

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Complexity Analysis and Simulation Results

- Hardware complexity of CN is studied using Quartus Prime synthesis tool.
- Fully-parallel implementation for a code rate $r = 5/6$ with $d_c = 12$ on Cyclone IV FPGA.
- FB-BRD algorithm reduces memory allocations by around 58% when compared with FB-EMS, and reduces computational complexity by around 15%.

Table 2: Synthesis Results for $d_c = 12$ on GF(64)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Logic Elements</th>
<th>Registers</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB-EMS ($n_m = 16$) [3]</td>
<td>109860</td>
<td>89940</td>
</tr>
<tr>
<td>FB-BRD ($n_{vc} = 4$, $n_B = 4$, $n_R = 3$)</td>
<td>94782</td>
<td>37308</td>
</tr>
</tbody>
</table>
Figure 7: Simulation Results over GF(64) with AWGN and BPSK Modulation
Figure 8: Simulation Results over GF(256) with AWGN and BPSK Modulation
FB-BRD decoder is based on the Forward-Backward (FB) EMS algorithm and the Best, Requested, and Default (BRD) algorithm.

- Allows variable nodes to request reliability of specific symbols from the CN.
- Adaptation of BRD with FB approach requires novel ECN called BRD-ECN.
- FB-BRD allows for reducing the global complexity of the decoder.

Synthesis results of the check node with $d_c = 12$ show a complexity reduction of at least 15% in favor of the BRD-FB CN compared to the FB-EMS CN in terms of logic elements and 60% reduction in terms of memory allocations.

Simulation results show no considerable performance loss for the FB-BRD tested at a FER down to $10^{-5}$ over different code rates and field orders.
References


Thank You :)  
Q & A?