The Best, the Requested, and the Default Elementary Check Node for EMS NB-LDPC Decoder

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Non-Binary LDPC Codes and Decoders

- NB-LDPC codes are extension of binary LDPC codes with $GF(q = 2^p)$ where $p > 1$ [1].
- Information block of size K symbols on GF(q) is encoded to a code block of size N symbols by adding M redundant symbols.
- Decoder consists of M Check Nodes (CNs) and N Variable Nodes (VNs).
- Each CN C_i is connected to d_c VNs, denoted as check degree of connectivity.
- Each VN V_j is connected to d_v CNs, denoted as variable degree of connectivity.

Non-Binary LDPC Codes and Decoders

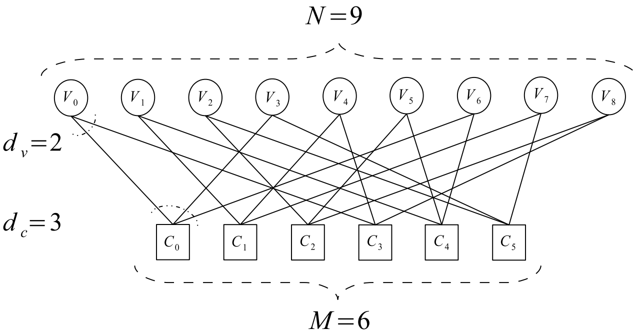

Figure 1: Tanner Graph of NB-LDPC Decoder

Non-Binary LDPC Codes and Decoders

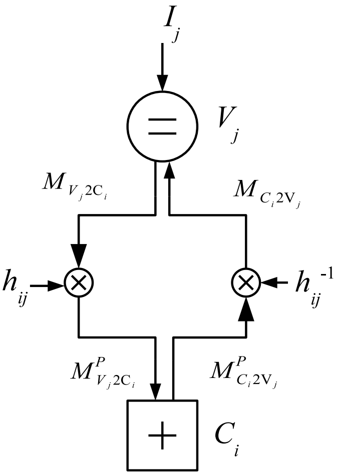

Figure 2: An Edge Connecting VN V_j to CN C_i.

Forward-Backward Extended Min-Sum Algorithm for NB-LDPC

- Complexity of NB-LDPC decoder is dominated in its CNs.
- Extended Min-Sum (EMS) is proposed in [2] to reduce complexity of CNs.
- How?
  - By truncating size of messages from q down to n_m.
  - Reducing check node operations from q^2 down to n_m^2.

Forward-Backward Extended Min-Sum Algorithm for NB-LDPC

- Forward-Backward approach [3] implements EMS algorithm by decomposing CN into 3 layers each of $d_c - 2$ Elementary Check Nodes (ECNs).

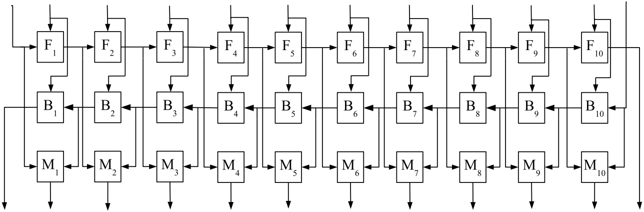

Figure 3: CN Decomposition in Forward-Backward Approach for $d_c = 12$

Forward-Backward Extended Min-Sum Algorithm for NB-LDPC

- Each ECN has only two inputs denoted as $(U^{\oplus}, U^{+})$ and $(V^{\oplus}, V^{+})$ each of size n_m.
- A matrix T_{Σ} is generated as

$$\begin{aligned} T_{\Sigma}^{+}[u][v] &= U^{+}[u] + V^{+}[v], \\ T_{\Sigma}^{\oplus}[u][v] &= U^{\oplus}[u] \oplus V^{\oplus}[v] \end{aligned} \tag{1}$$

  $T_{\Sigma}^{\oplus}$ and T_{Σ}^{+} correspond to vector of GF symbols and their LLR values respectively.
- ECN generates n_m candidates (GF and LLR couples) sorted in descending order of their reliability.

The Best, Requested, and Default Algorithm

- The Best, Requested, and Default (BRD) algorithm [4] is generic NB-LDPC decoding algorithm.
- Allows VNs to request specific symbols from CNs.
- Requested symbols preserve decoding performance with shorter message sizes.
- Uses compression and decompression block at each side of edge.

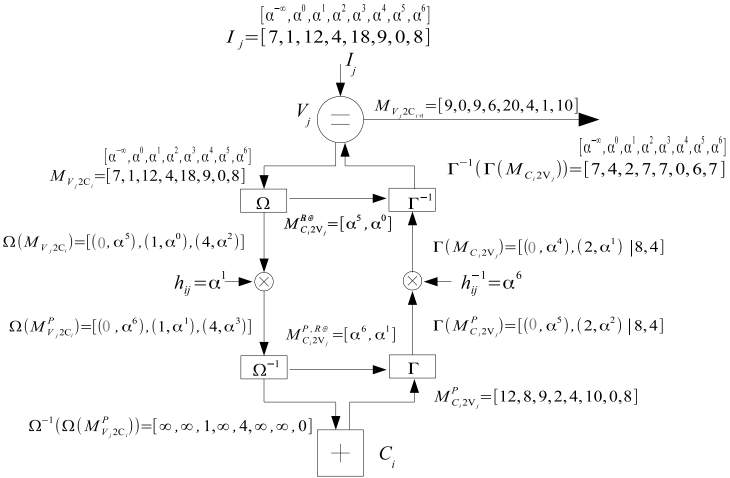

Figure 4: Toy Example of BRD Decoder on GF(8) with $n_{vc} = 3$, $n_B = 2$ and $n_R = 2$

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Forward-Backward BRD Decoder

- For BRD algorithm to be compatible with FB algorithm, a variant ECN is needed, called BRD-ECN.
- BRD-ECN composed of two sub-blocks and three input vectors as shown

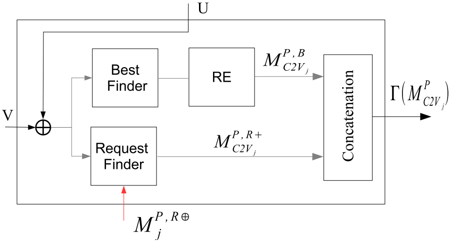

Figure 5: Structure of BRD-ECN

Forward-Backward BRD Decoder

- White ECN blocks → Conventional ECNs as in [3].
- Grey ECN blocks → BRD-ECNs.

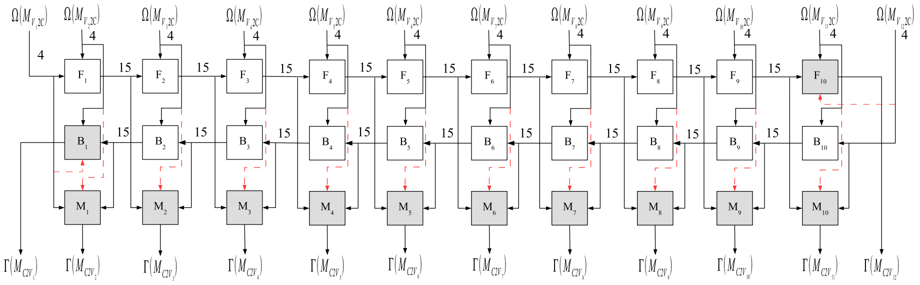

Figure 6: Forward-Backward BRD Architecture for $d_c = 12$

Complexity Analysis and Simulation Results

- Integrating BRD algorithm with FB algorithm reduces complexity of CN and VN units by reducing
  - Communication load.
  - Sorters size.
  - Arithmetic operations (real additions, GF additions, GF multiplications).
  - Memory Allocations.

Complexity Analysis and Simulation Results

Table 1: Size of Exchanged Messages per Edge on GF(64)

Scheme	Code Rate	Inputs		Outputs	
		$n_{vc}^{\oplus}$	n_{vc}^{+}	$n_{cv}^{\oplus}$	n_{cv}^{+}
FB-EMS[3]	any	20	19	20	19
FB-BRD	$r \geq 5/6$	4	3	4	6
	$r = 1/2$	8	7	6	10
	$r = 1/3$	13	12	7	14

Complexity Analysis and Simulation Results

- Hardware complexity of CN is studied using Quartus Prime synthesis tool.
- Fully-parallel implementation for a code rate $r = 5/6$ with $d_c = 12$ on Cyclone IV FPGA.
- FB-BRD algorithm reduces memory allocations by around 58% when compared with FB-EMS, and reduces computational complexity by around 15%.

Table 2: Synthesis Results for $d_c = 12$ on GF(64)

Scheme	Logic Elements	Registers
FB-EMS ($n_m = 16$) [3]	109860	89940
FB-BRD ($n_{vc} = 4, n_B = 4, n_R = 3$)	94782	37308

Complexity Analysis and Simulation Results

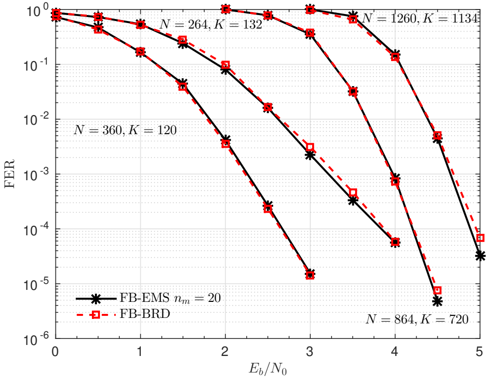

Figure 7: Simulation Results over GF(64) with AWGN and BPSK Modulation

Complexity Analysis and Simulation Results

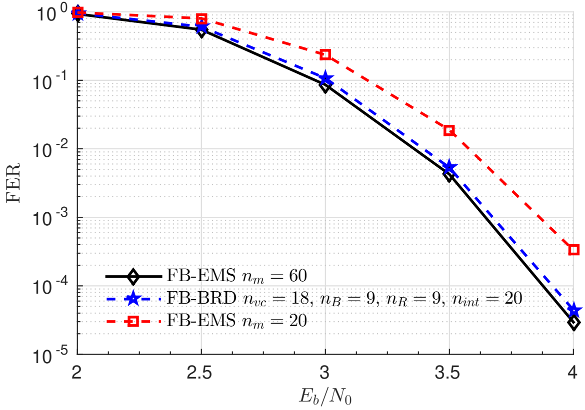

Figure 8: Simulation Results over GF(256) with AWGN and BPSK Modulation

Conclusion

- FB-BRD decoder is based on the Forward-Backward (FB) EMS algorithm and the Best, Requested, and Default (BRD) algorithm.
- Allows variable nodes to request reliability of specific symbols from the CN.
- Adaptation of BRD with FB approach requires novel ECN called BRD-ECN.
- FB-BRD allows for reducing the global complexity of the decoder.
- Synthesis results of the check node with $d_c = 12$ show a complexity reduction of at least 15% in favor of the BRD-FB CN compared to the FB-EMS CN in terms of logic elements and 60% reduction in terms of memory allocations.
- Simulation results show no considerable performance loss for the FB-BRD tested at a FER down to 10^{-5} over different code rates and field orders.

References

[1] M. Davey and D. MacKay, "Low-density parity check codes over GF(q)," *IEEE Communications Letters*, vol. 2, no. 6, pp. 165–167, 1998. DOI: 10.1109/4234.681360.

[2] D. Declercq and M. Fossorier, "Decoding Algorithms for Non-Binary LDPC Codes Over GF(q)," *IEEE Transactions on Communications*, vol. 55, no. 4, pp. 633–643, 2007.

[3] A. Voicila, D. Declercq, F. Verdier, M. Fossorier, and P. Urard, "Low-complexity decoding for non-binary LDPC codes in high order fields," *IEEE Transactions on Communications*, vol. 58, no. 5, pp. 1365–1375, 2010. DOI: 10.1109/TCOMM.2010.05.070096.

[4] J. Jabour, C. Marchand, and E. Boutillon, "The best, the requested, and the default non-binary LDPC decoding algorithm," in *International Symposium on Topics in Coding 2021*, 2021.

Thank You :)

Q & A?