AI4CODE

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C4-Sequences: Rate Adaptive Coded Modulation for Few Bits Message

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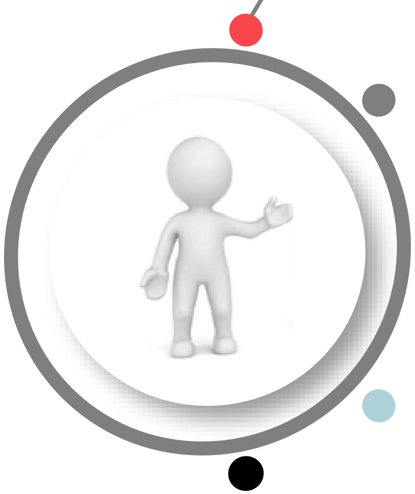
Outline



Summary of Marchand's presentation

"Sting art" sequences

C4 sequences

Geometrical shaping

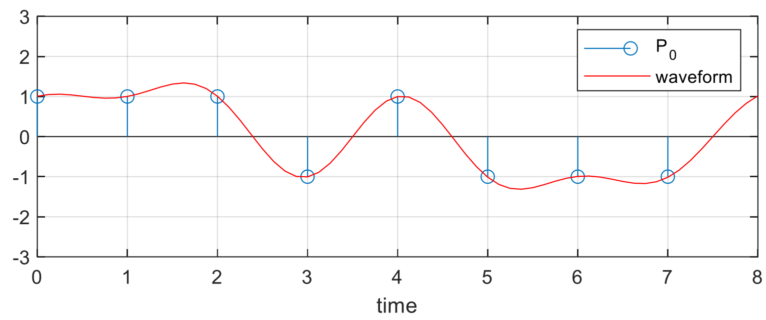
Coded modulation

Binary Cyclic Code Shift Keying modulation

P_0 = 11101000 + BPSK modulation, q = 8, m = 3 bits per CCSK symbol.

- CCSK modulation:
  - P_0 = 11101000
  - P_1 = 01110100
  - P_2 = 00111010
  - P_3 = 00011101
  - P_4 = 10001110
  - P_5 = 01000111
  - P_6 = 10100011
  - P_7 = 11010001

Code rate 3/8



Binary message : 011001100

Make 3-uplet symbols: $(011)_2(001)_2(100)_2$

Take decimal value: 3 1 4

Associate CCSK symbol P_3 P_1 P_4

Send => 000111010111010010001110

[1] G.M. Dillard et al. « Cyclic code shift keying: a low probability of intercept communication technique.” IEEE Trans. on Aerospace and Electronic Systems, 39(3):786–798, 2003.

More about CCSK

Element of the coding sequence are called “chip”

CCSK not restricted to BPSK modulation [1]:

8-PSK, 16-QAM, 64-APSK can be used to encode the chips.

Truncate the CCSK sequence to increase the code rate [1]:

Example: send only first l = 5 chips, instead of the whole sequence.

Binary message : 011001100

Make 3-uplet symbols: $(011)_2(001)_2(100)_2$

Take decimal value: 3 1 4

Associate CCSK symbol P_3 P_1 P_4

Send => 000111010111010010001110

- => 00011- - -01110- - -10001- - -
- => 000110111010001

Code rate 3/5

[1] G.M. Dillard et al. « Cyclic code shift keying: a low probability of intercept communication technique.” IEEE Trans. on Aerospace and Electronic Systems, 39(3):786–798, 2003.

What is a good CCSK sequence?

- Notation: $\mathbf{x} = (x(0), x(1), \dots, x(q-1))$ a complex CCSK sequence with $\|\mathbf{x}\|^2 = q$ (energy of 1 in average per chip).
- From $\mathbf{x}$ => define a codebook of *q* sequences $\mathbf{x}_a = (x(a)\ x(a+1), \dots, x(a+q-1))$

(here, left rotation, with index operation done modulo q).

Minimum distance of the code: $D_m^2 = \min_{a \neq b} \| \mathbf{x}_a - \mathbf{x}_b \|^2$

$$\| \mathbf{x}_a - \mathbf{x}_b \|^2 = \| \mathbf{x}_a \|^2 + \| \mathbf{x}_b \|^2 - 2\, \mathrm{Real}(< \mathbf{x}_a , \mathbf{x}_b >).$$

Thus : $D_m^2 = 2q - 2 \max(\mathrm{Real}(< \mathbf{x}_a , \mathbf{x}_b >, a \neq b).))$

What's about truncated CCSK sequences?

- How evolve the minimum distance when truncated sequences are used?

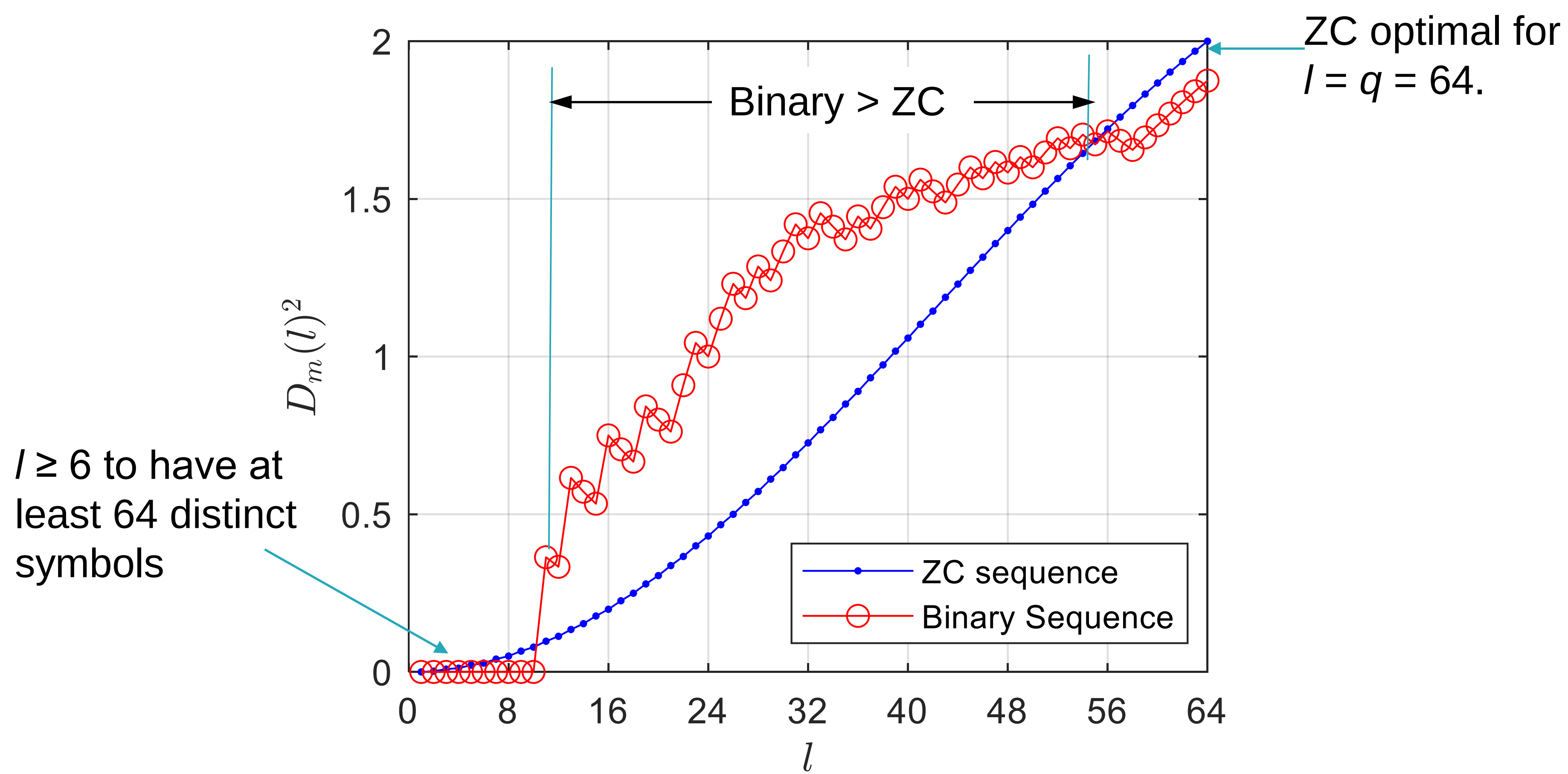
- Convenient to use a normalized distance, i.e., a distance per chip, i.e.

$$D_m(l)^2 = \frac{1}{l}\min\left(\left\|\mathrm{x}_a^l - \mathrm{x}_\mathrm{b}^\mathrm{l}\right\|^2, a \neq b\right)$$

- With x_a^l the length-*l* truncated sequence of x_a , i.e.,

$$\mathrm{x}_a^l = (x(a), x(a+1), \dots, x(a+l-1))$$

Evolution of $D_m(l)$ as a function of the truncation length?



- It is possible to do better?

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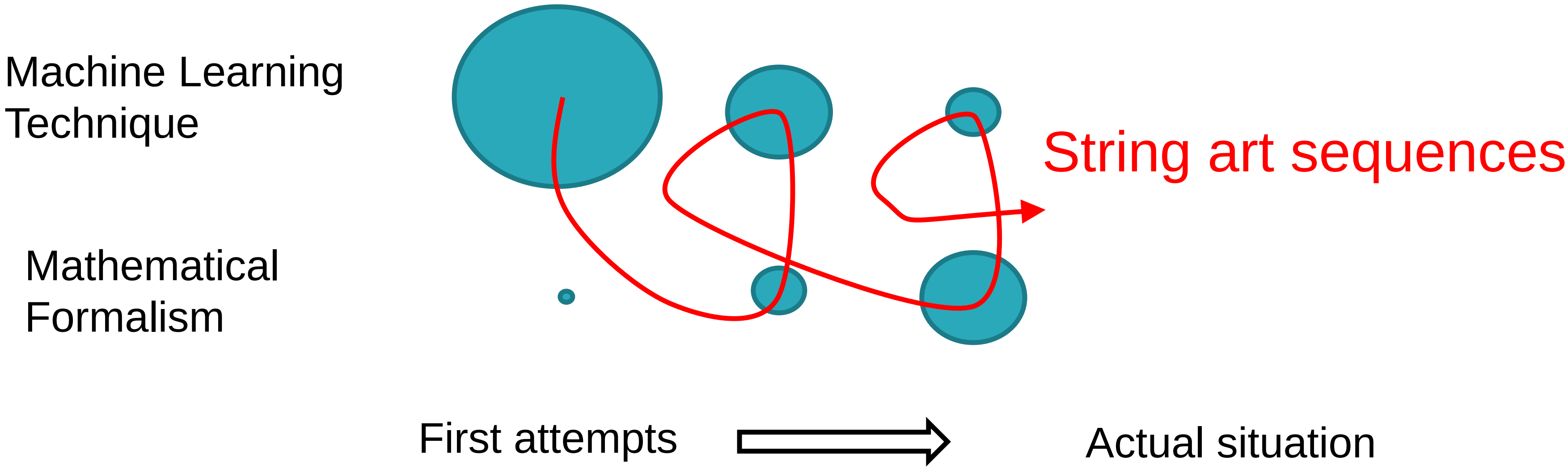
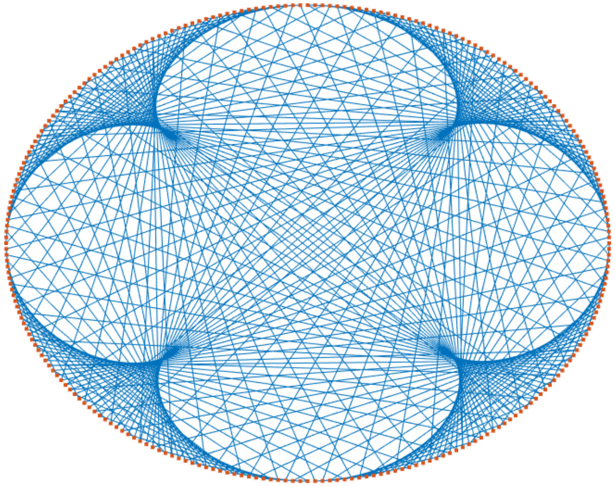
Optimization of NB-CCSK sequences

- Multi-objective optimization problem: maximize D_l^2 for $l = 1, \ldots, q$.

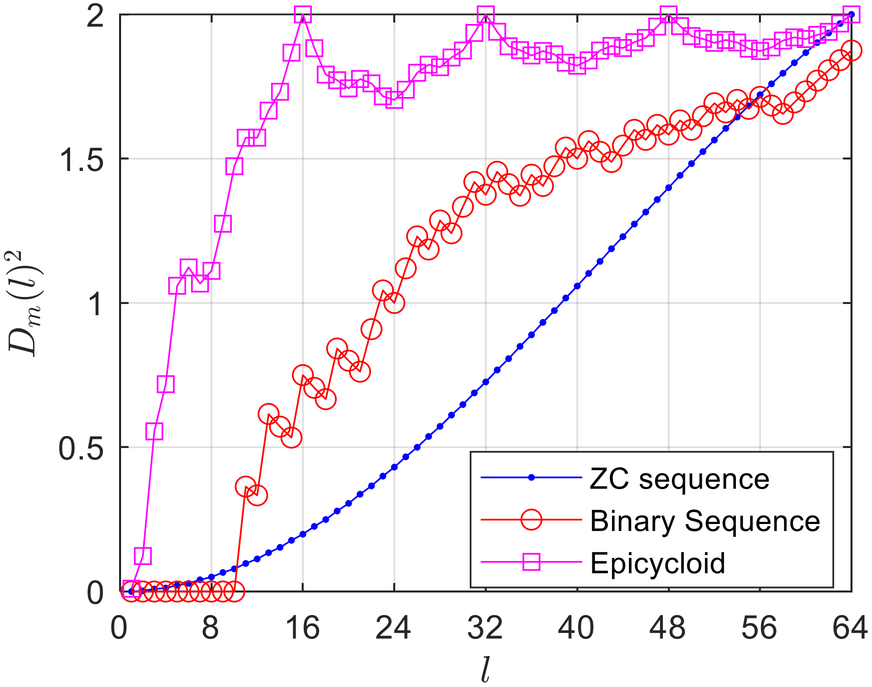


- Set of optimal solutions obtained.

Set of solutions with $D_m(l) = 2$ for several values of l.



- $\pi(i+1)=\mathrm{mod}(5\pi(i)+1, q)$



Optimal for l = q/4, q/2, 3q/4 and q.
Always above Binary and ZC sequences…

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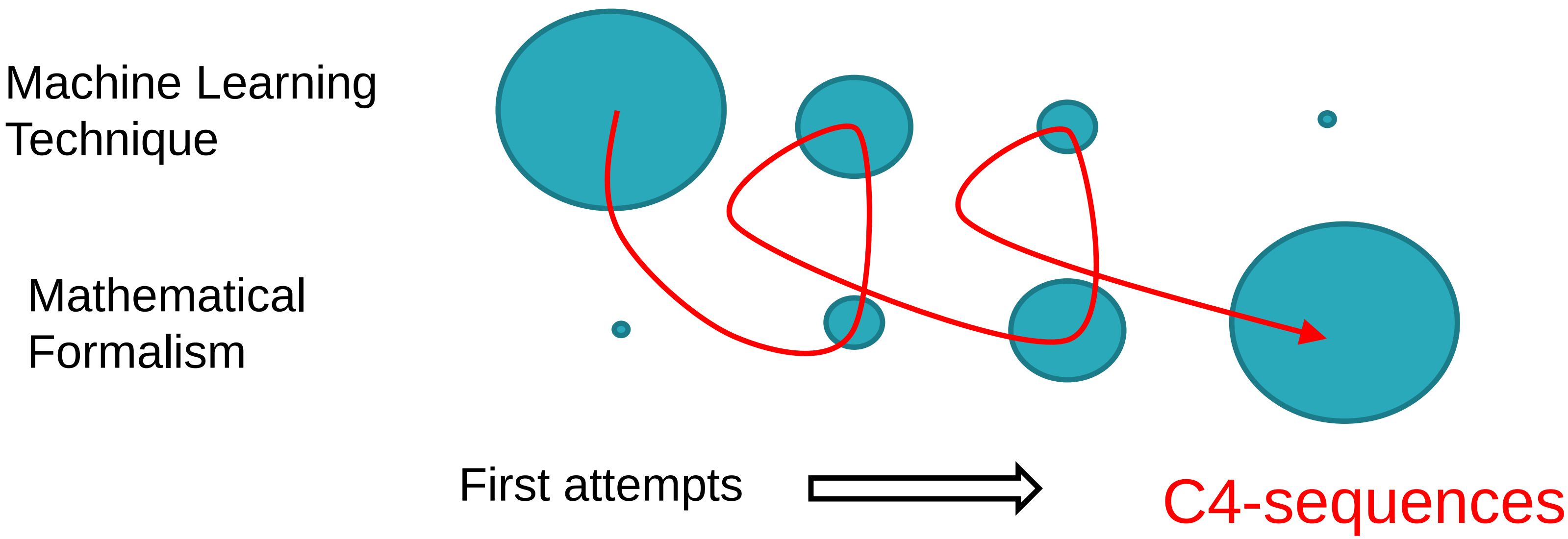
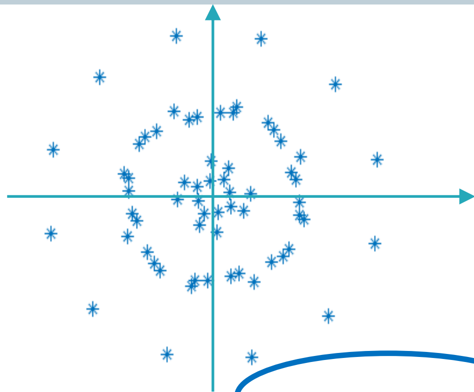
- Multi-objective optimization problem: maximize D_l^2 for l = 1, …, q.



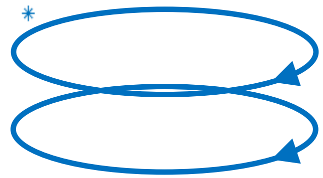
- Set of optimal solutions obtained.

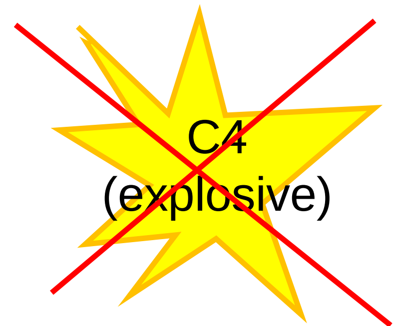
What does C4 means?

- Constellation



- Circular





- (auto)-Correlation

$$\sum_{n=0}^{q-1} x(n)x(n+\tau)'$$

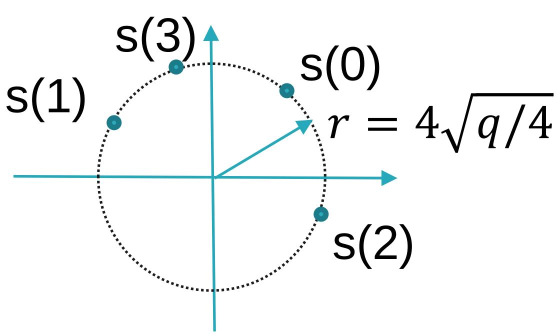
- Cross

(celtic cross)

Mathematical recipe for C4-sequence

Generation of a C4 sequence of length $q = 4p$ in 3 steps.

1) Select randomly $q/4$ points
   on a circle of radius $r = 4\sqrt{q/4}$



2) From length *q/4* vector ***S***, generate length *q* vector **X** as

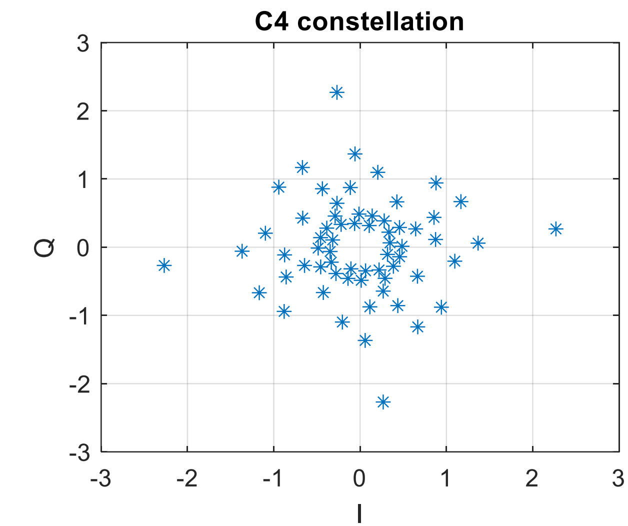
   X = [0 0 0 S(0) 0 0 0 S(1) …. 0 0 0 S(*q*/4-1)]

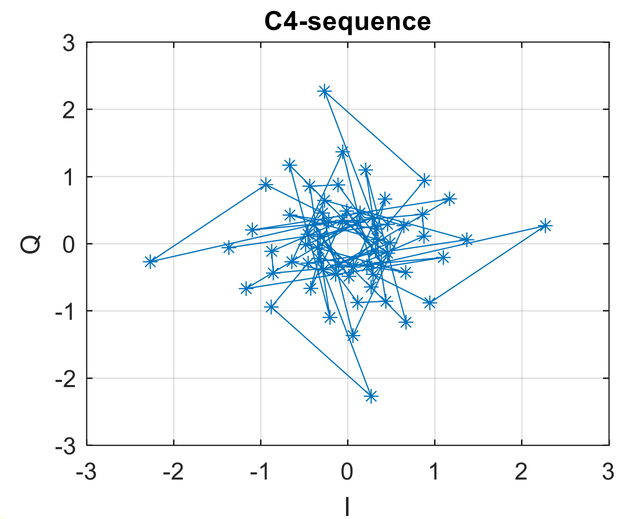
3) Compute the Inverse Discrete Fourier Transform **x** of **X**, **x** = IDFT(**X**).

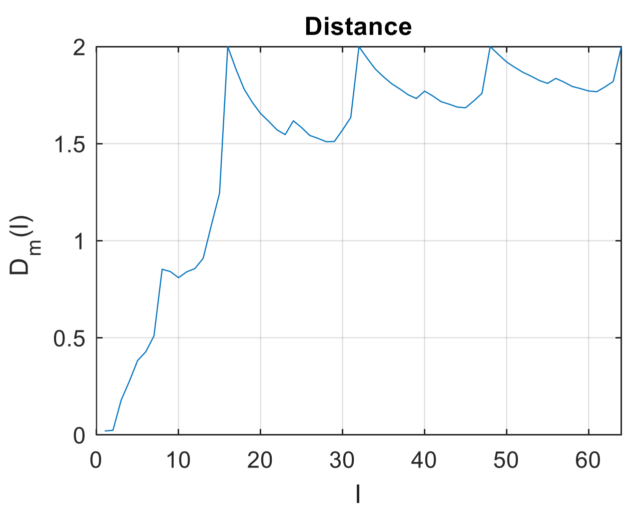
```
Function x = random_C4(q);

% Step 1
theta = 2*pi*rand(1,q/4);
S = 4*sqrt(q/4)*exp(1i*theta);

% Step 2
X = kron(S, [0 0 0 1]);

% Step 3
x = ifft(X);
```

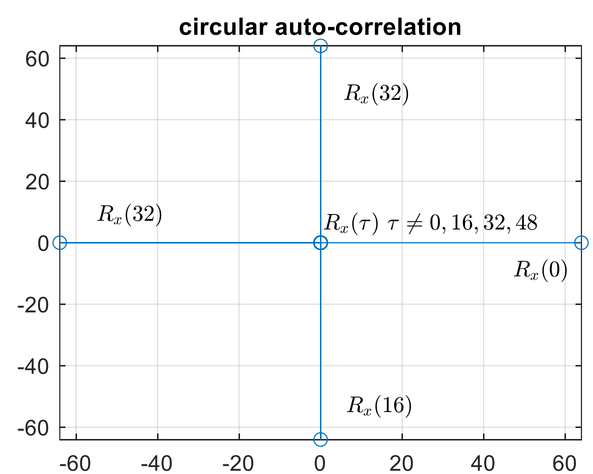
…and x is a C4 sequence

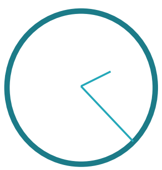
Experimentation Results q = 64











Clockwise rotation

Clock-wise and Anti-clockwise C4 sequences

```
q = 64;
theta1 = rand(1,q/4);
S1 = 4*sqrt(q/4)∗ exp(2∗pi∗1i*theta1);
X = kron(S1, [0 0 0 1]);
x = ifft(X);
```

Clock-wise C4 sequence

```
theta2 = rand(1,q/4);
S2 = 4*sqrt(q/4)∗ exp(2∗pi∗1i*theta2);
Y = kron(S2, [0 1 0 0]);
y = ifft(Y);
```

Anti-clockwise C4 sequence

Theorem: if x is a clockwise C4-sequence and y and anti-clockwise C4-sequence, then,

for all a, b, $<x_a, y_b> = 0$.

i.e., the two sets of sequences are orthogonal.

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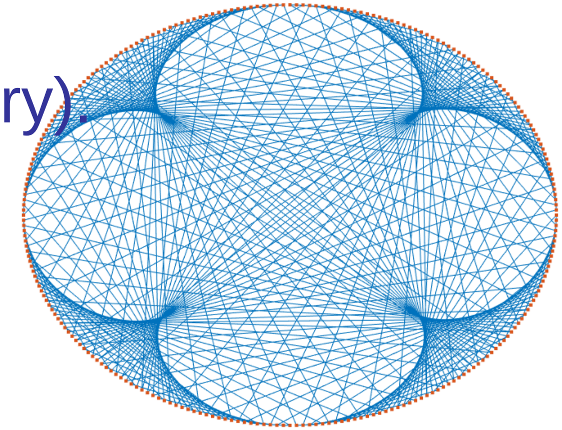
Properties of C4 sequences



- Invariant by a π/2 rotation (4-fold symmetry).
- $x\left(n+\frac{q}{4}\right)=j^{-c}x(n)$, c=1 clockwise,
  c=-1, anticlockwise.
- $R_x(\tau)=j^{-kc}q, \text{when } \tau=\frac{kq}{4}$, 0 otherwise
- $D_l\left(k\frac{q}{4}\right)=2, k=1,2,3,4$
- From a length q C4 sequence, it is possible to generate a set of q lengths sequence of length p = q/4 $\{\mathrm{x}_a^p\}, a=0,1,\ldots,q$ verifying
$$\left\|\mathrm{x}_a^p-\mathrm{x}_b^p\right\|^2 \geq q/2 \text{ if } a \neq b$$

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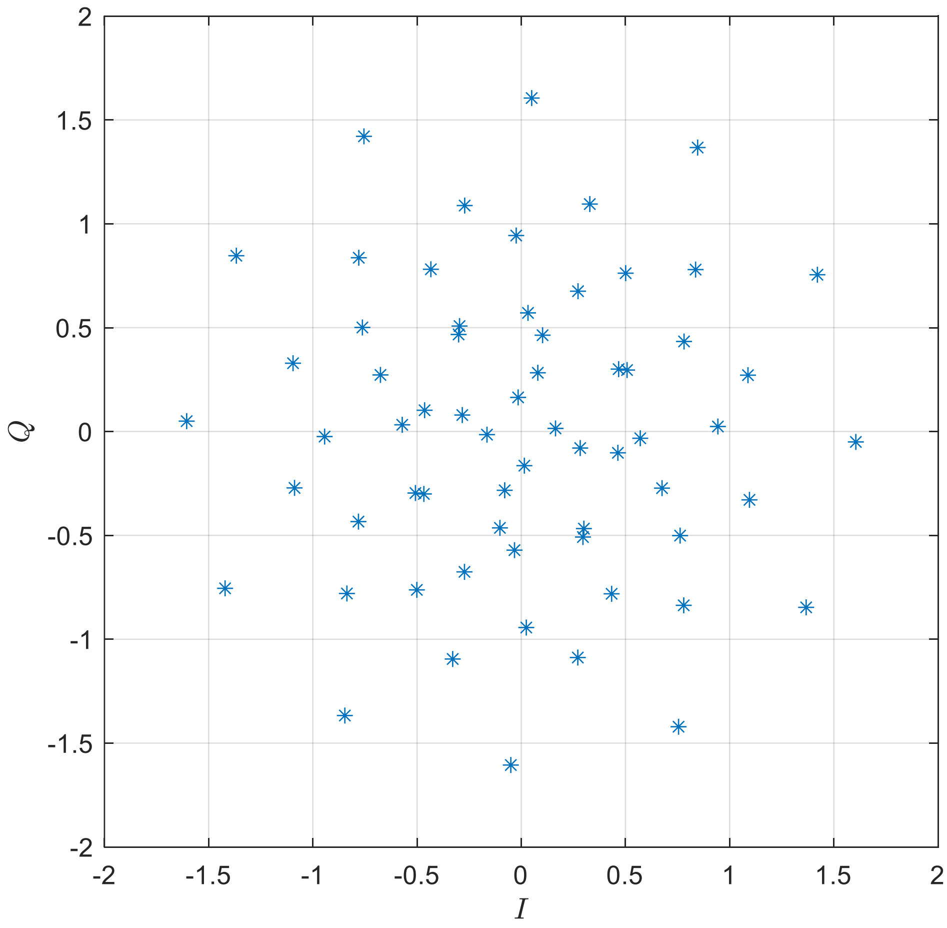
Shaping of C4-Constellations

- String art sequences are particular case of C4-sequences.
- By the selection of vector θ, flexibility to shape the C4-sequence.
  - => points on unit circle
  - => points on 2 or more circles
  - => Optimization for a given criteria
    - Mutual information (MI) of the constellation alone in AWGN channel.

q = 64 C4-constellation, SNR of 10 dB

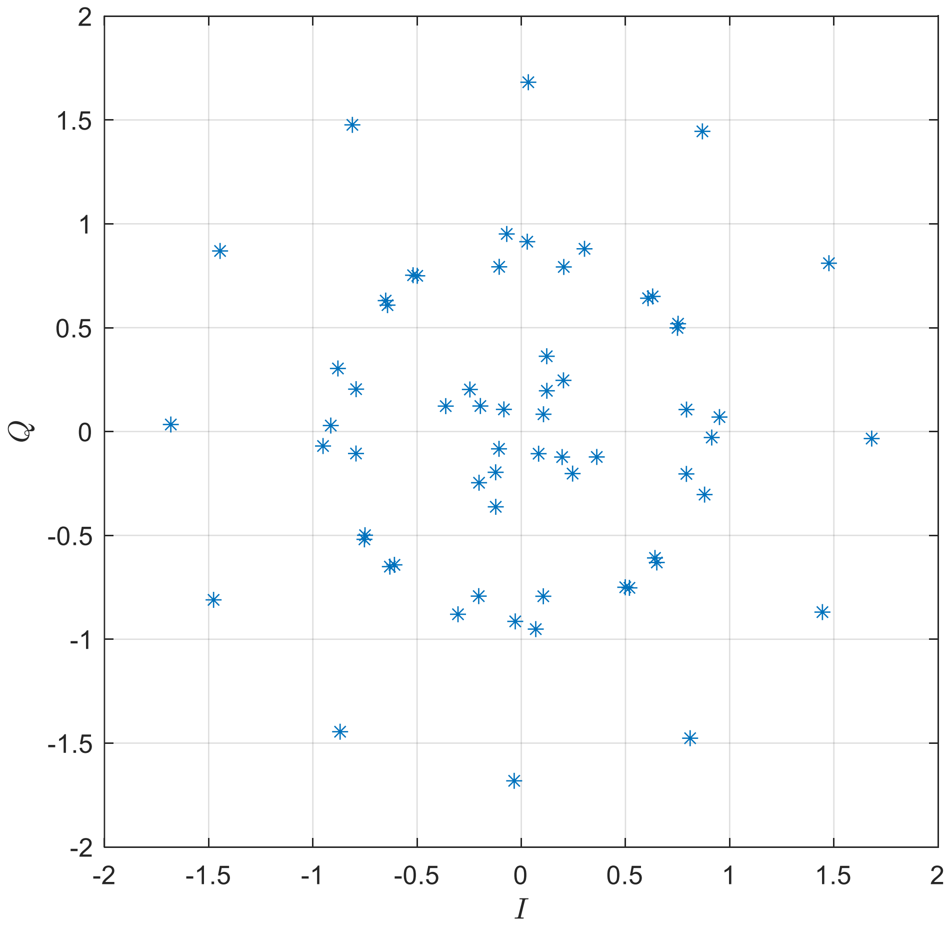


Capacity: 3.4594 bit/s/Hz

MI: 3.4192 bit/s/Hz

98.8379 % of capacity

q = 64 C4-constellation, SNR of 5 dB

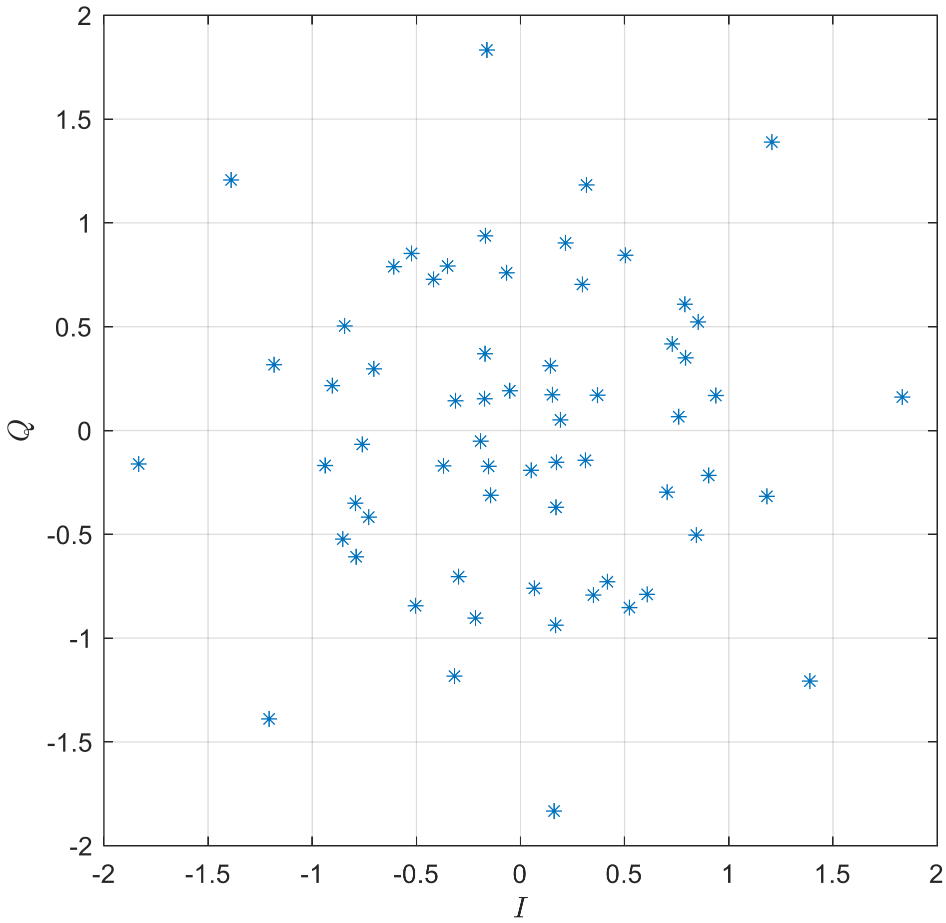


Capacity: 2.0574 bit/s/Hz

MI: 2.0537 bit/s/Hz

99.8227 % of capacity

q = 64 C4-constellation, SNR of 0 dB

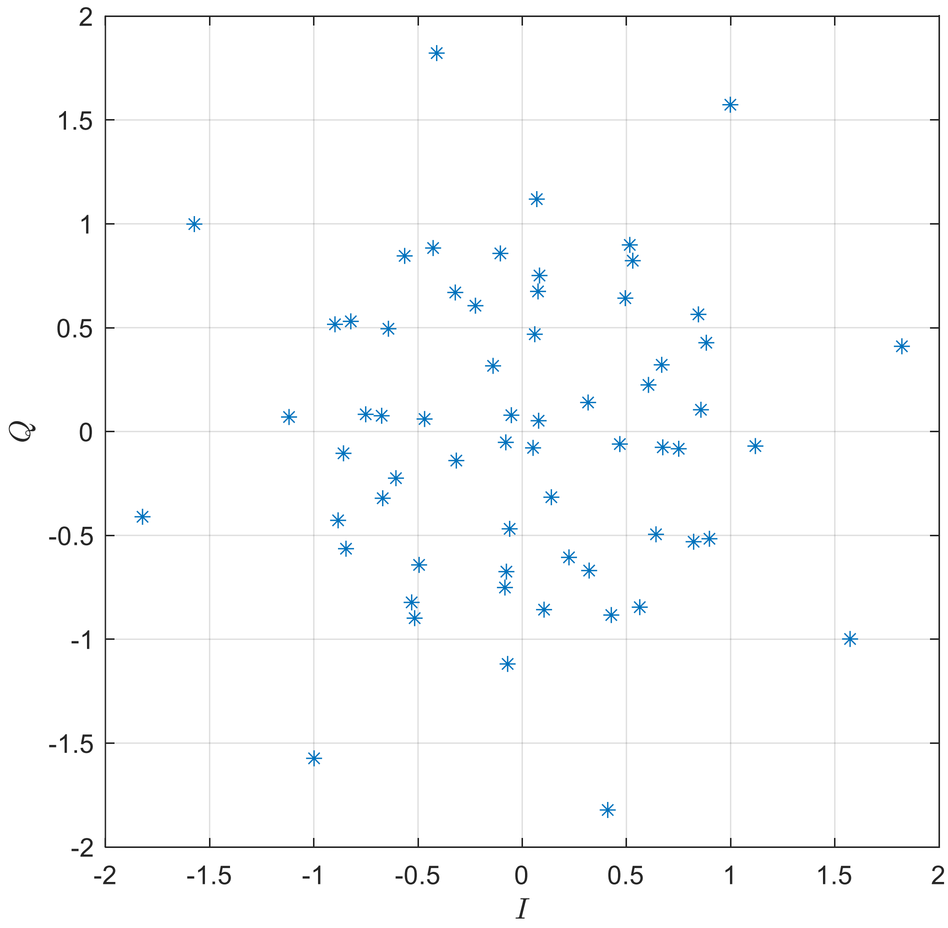


Capacity: 1.0000 bit/s/Hz

MI: 0.9998 bit/s/Hz

99.998 % of capacity

q=64 C4-constellation, SNR of -5 dB

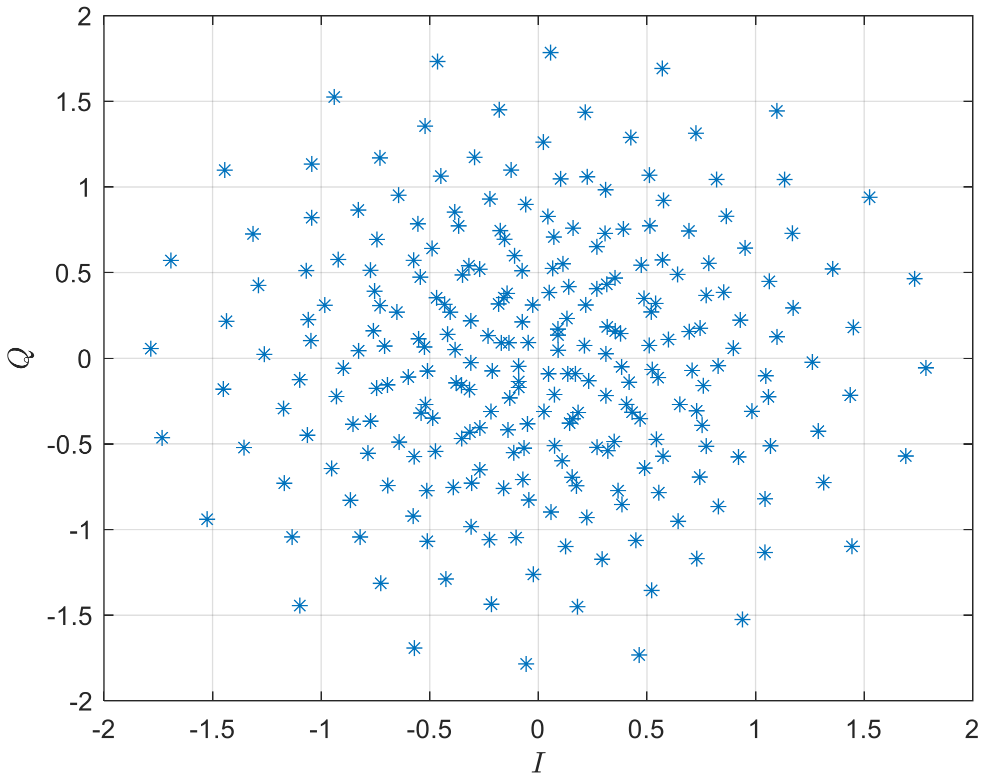


Capacity: 0.396409 bit/s/Hz

MI: 0.396408 bit/s/Hz

99.999891 % of capacity

q=256 C4-constellation, SNR of 15 dB



Capacity: 5.0278 bit/s/Hz

MI: 4.9867 bit/s/Hz

99.18238184 % of capacity

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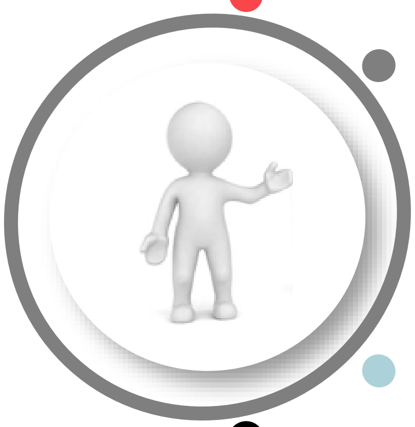

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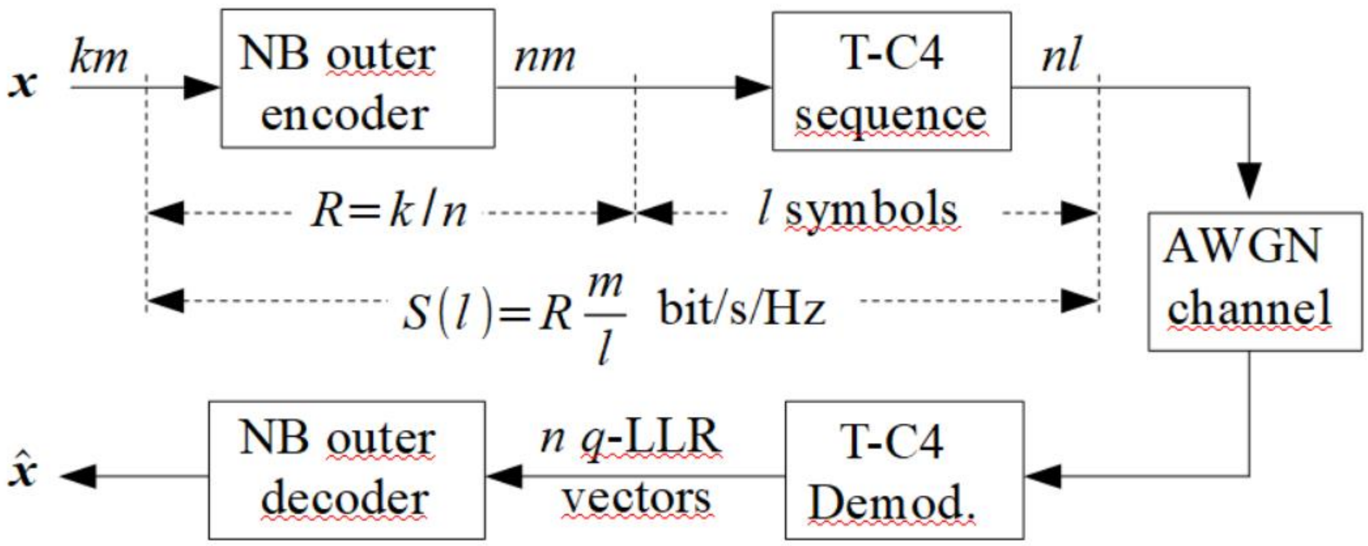
Proposed coded modulation ($q = 2^m$)

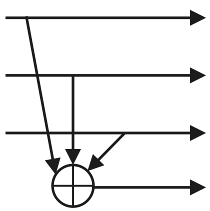


- Global spectral efficiency: $\frac{mR}{\bar{l}}$ bit/s/Hz.

Degree 4 Parity Check over GF(64)

q = 64, m = 6 C4 sequence optimized for 5 dB of SNR

18 bits



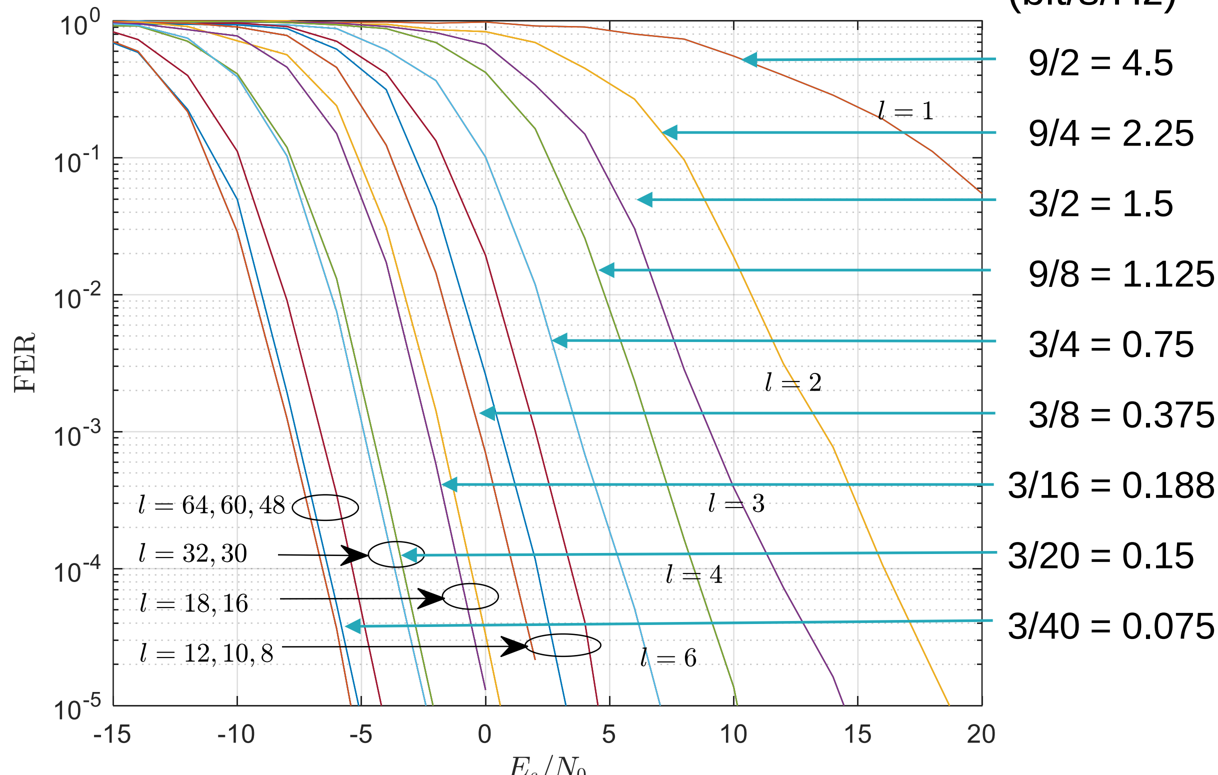
24 bits

Outer NB code: single parity over GF(64), R = 3/4

Spectral efficiency:

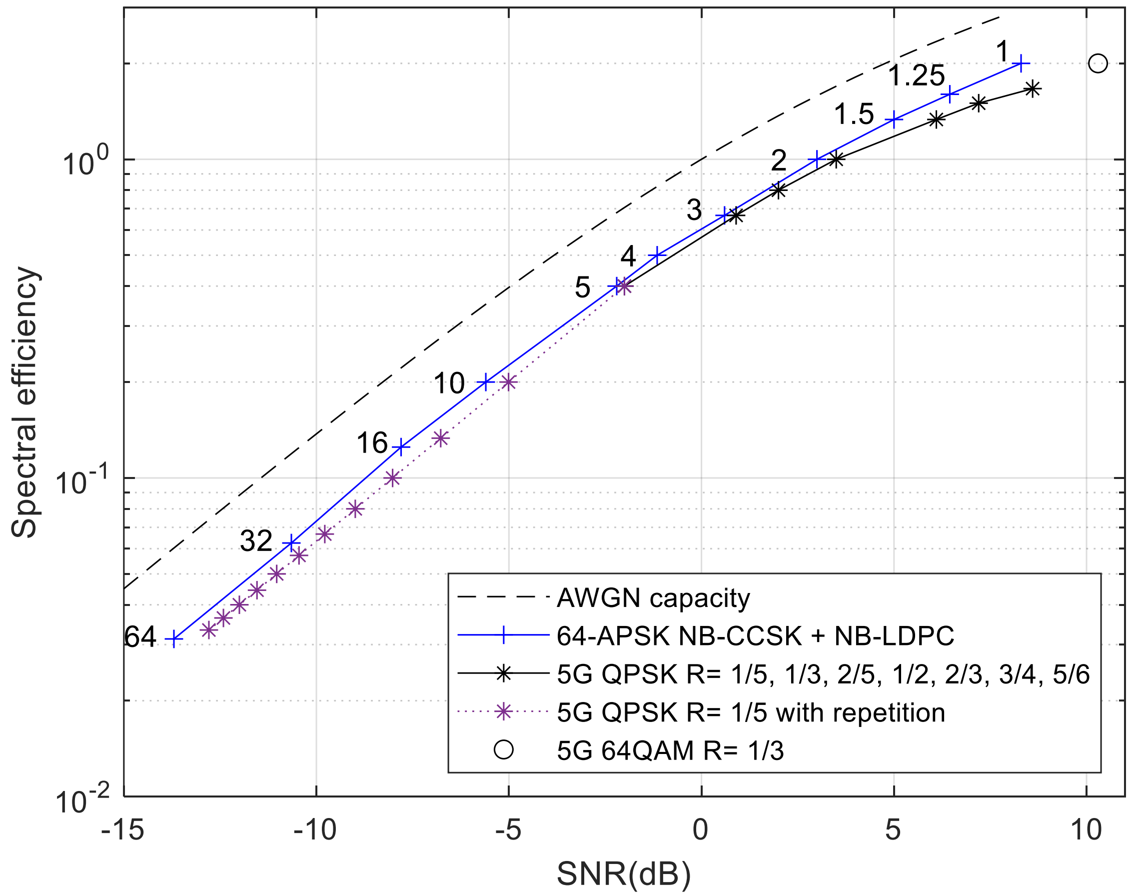
$\frac{mR}{l} = \frac{6\times 3/4}{l} = \frac{9}{2l}$ bit/s/Hz.

Overall spectral efficiency (bit/s/Hz)



0.07 bit/s/Hz

Finite length simulation, q = 64, k = 120 bits, NB-LDPC R = 1/3.



FER 10^{-3}

- Finite length simulation confirmed theoretical results.

Potential applications of C4-sequences

- May be used for geometrical shaping
- May be used in multi-users environment to identify users.
- May be used, like ZC sequences, for synchronization.
- May be used for flexible rate adaptive coded modulation.

Potential extensions

- The 4 in C4 can also be the 4 optimal truncation lengths.

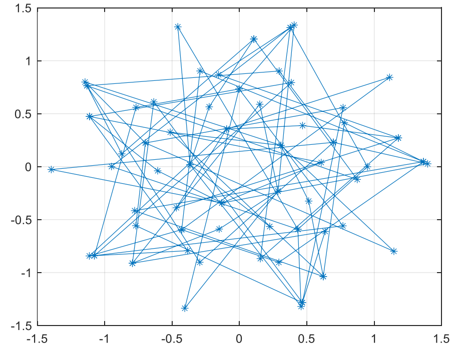
- Extension of C4 sequences to C3 or C5 sequences possible: in the construction process, replace

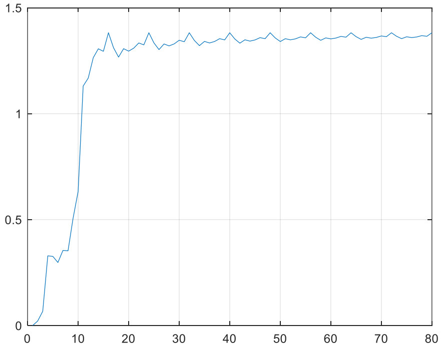
  Y = 4*sqrt(q/4)∗kron(S, [0 1 0 0]);

  by

  Y = 5*sqrt(q/5)∗kron(S, [0 1 0 0 0]); (for example)

  to create C5 sequence with a 5-fold symmetry





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Thank you !