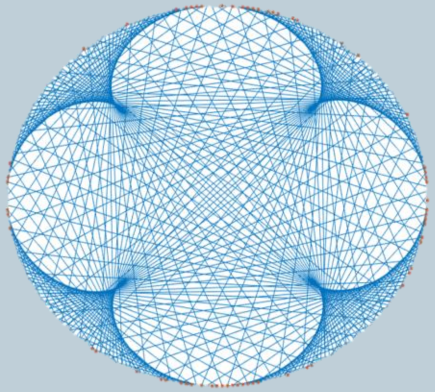
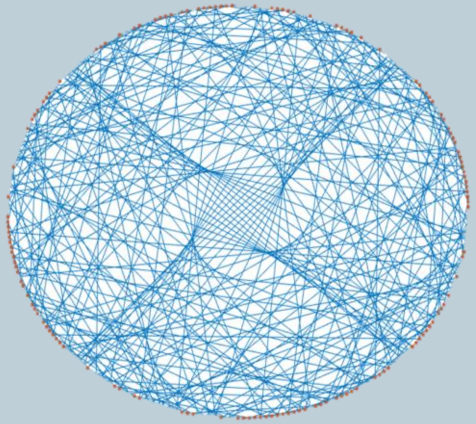
Rate-Adaptive Cyclic Complex Spreading Sequence for Non-Binary Decoders

ISTC 2023

Cédric Marchand
Alexandru-Liviu Olteanu
Emmanuel Boutillon



12th International Symposium on Topics in Coding



AI4CODE

6 September 2023, Brest

Introduction: a solution to rate-adaptive code

- Decoder must adapt to channel conditions
  - Costly Rate-adaptive decoder architecture
  - Plenty of codes f(K, R)
  - Plenty of Modulations
  - Sub-optimal Repetition for very low rate

→

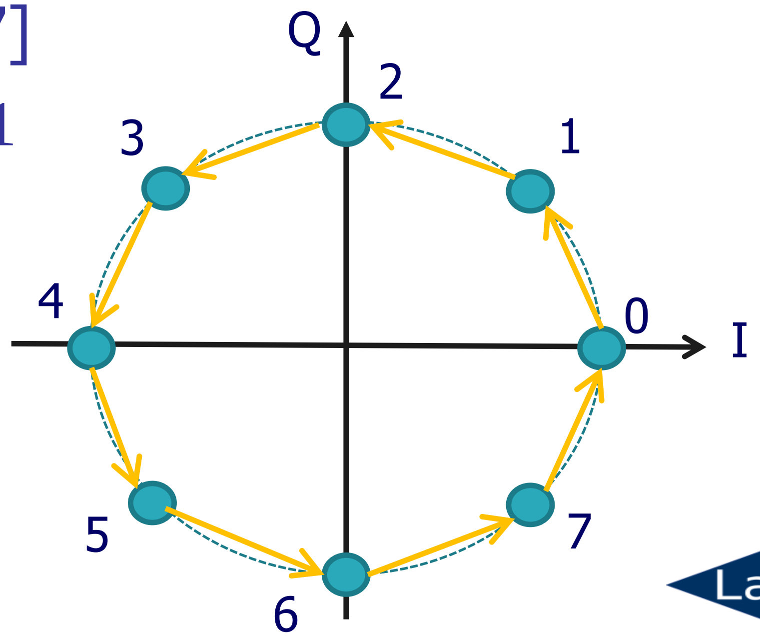
- Solution
  - 1 NB decoder
    - optimized for 1 code rate
  - 1 Modulation
  - 1 Inner code responsible of Rate matching
    - The Punctured NB-CCSK

Example of NB-CCSK with 8-PSK

- The CCSK modulation spread a symbol $b, b \in \{0, \ldots, q-1\}$ on a sequence $\mathbf{G}_b$ defined as the circular left shift of the root sequence $\mathbf{G}_0$ by b positions.
- A sequence of 8 chips, each chip mapped on a distinct point of a 8-PSK modulation
- sequence $\mathbf{G}_0$ =[0, 1, 2, 3, 4, 5, 6, 7]
- PSK constellation $C(i) = e^{j2\pi ji/q}$, $j^2 = -1$
- $C(\mathbf{G}_0(i)) = e^{j(2\pi \mathbf{G}_0(i)/q)}$, $j^2 = -1$

Codeword table

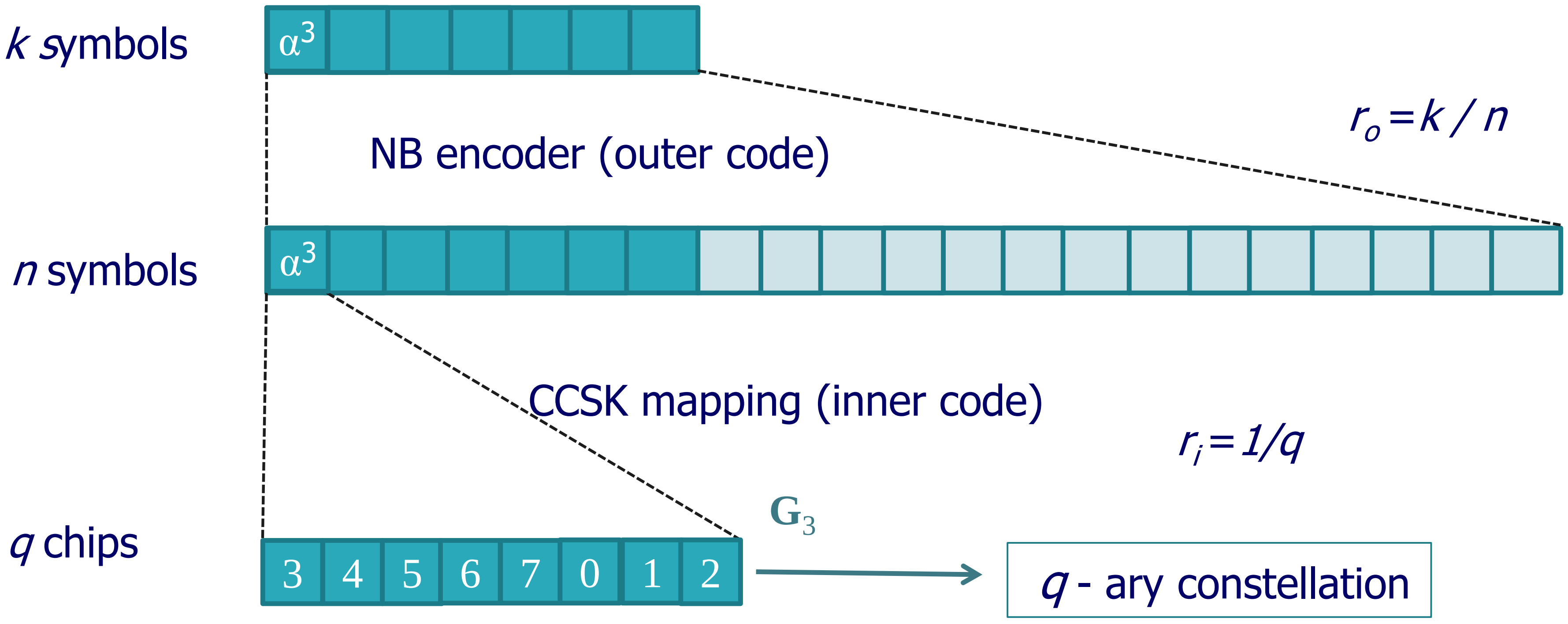
b	$\mathbf{G}_b$
0	0 1 2 3 4 5 6 7
1	1 2 3 4 5 6 7 0
2	2 3 4 5 6 7 0 1
3	3 4 5 6 7 0 1 2
4	4 5 6 7 0 1 2 3
5	5 6 7 0 1 2 3 4
6	6 7 0 1 2 3 4 5
7	7 0 1 2 3 4 5 6



Mapping of G_0 on 8-PSK constellation

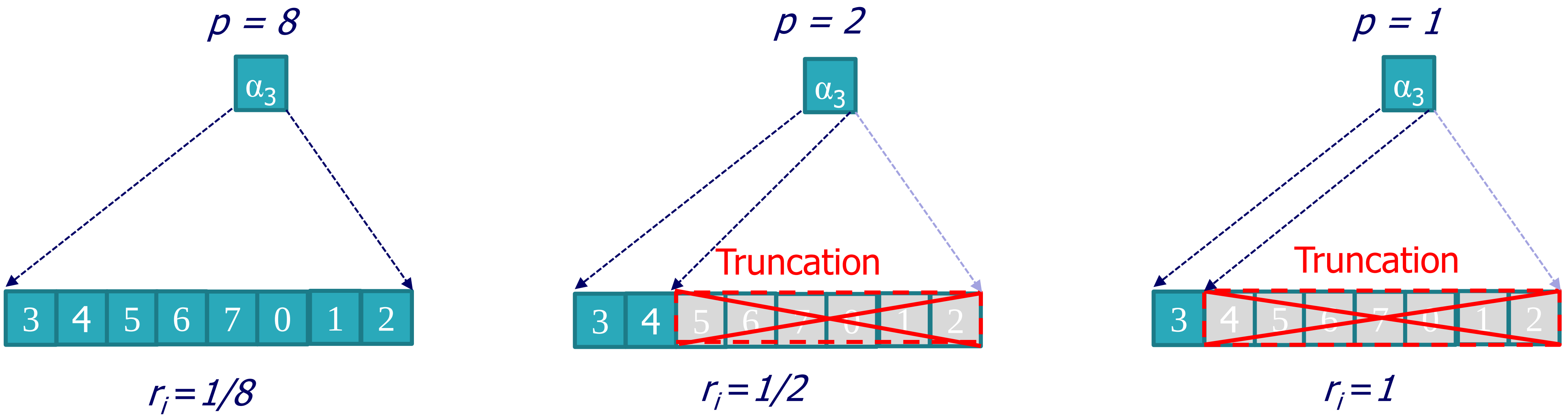
Inner CCSK code: principle

Non binary symbols are in GF(q) with $m = \log_2(q)$ bits (m bits per symbol)



NB-CCSK: Truncation

- Truncation from q to p chips to make CCSK rate-adaptive [1][2]

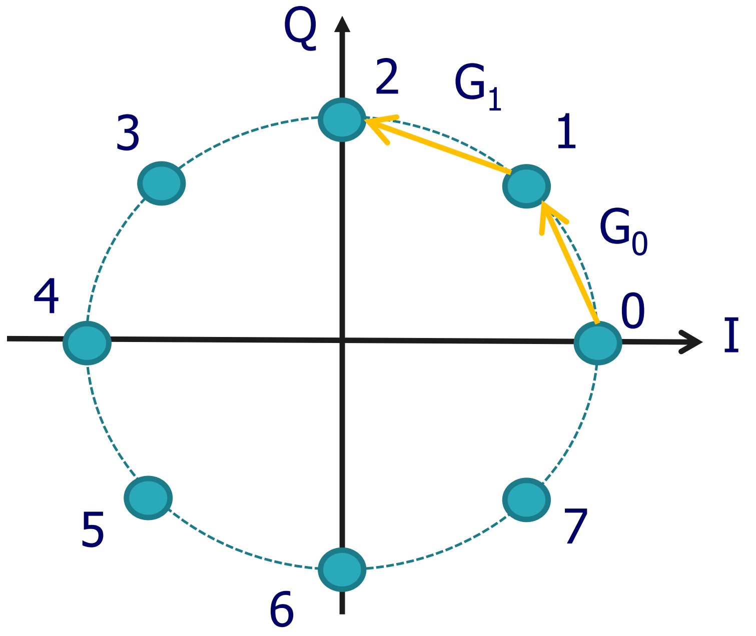


- Per symbol puncturing[2] :Half symbols with p =1, the other half with p = 2
  - p =1.5, r_i=2/3

[1] "Cyclic Code Shift Keying: A Low Probability of Intercept Communication Technique", 2003
[2] C. Marchand and E. Boutillon, "Rate-adaptive Inner Code for Non-Binary Decoders," 2021

Example: NB- CCSK mapped on 8-PSK with *p* =2

- G_0=[0, 1, 2, 3, 4, 5, 7], *p* =2
- Squared Euclidian Distance between G_0 and G_1
  - $d(G_0, G_1)^2 = ||C(1) - C(0)||^2 + ||C(2) - C(1)||^2$
  - $d(G_0, G_1)^2 = 1{,}17$
- Minimum squared distance of the code
  - $D(G) = \min_{a \neq b}\{d(G_a, G_b)^2\}$
  - $D(G)=1{,}17$



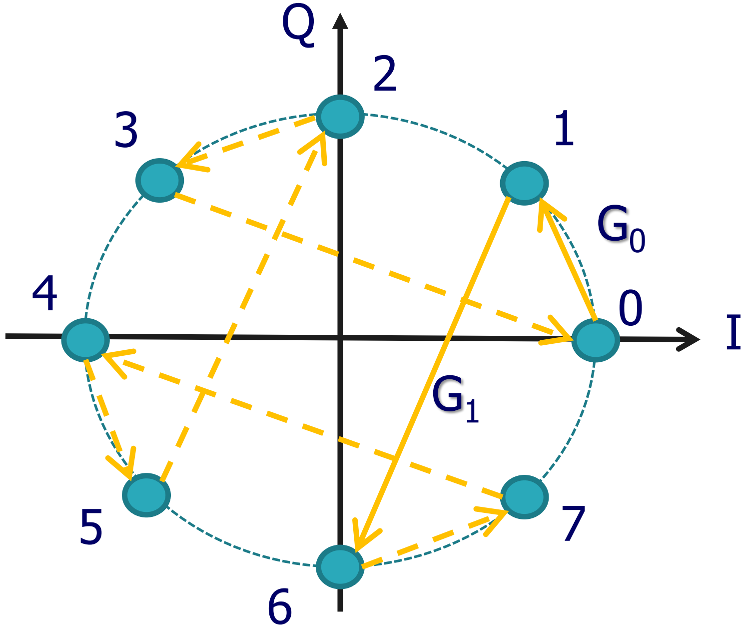
b	G_b
0	0 1 2 3 4 5 6 7
1	1 2 3 4 5 6 7 0
2	2 3 4 5 6 7 0 1
3	3 4 5 6 7 0 1 2
4	4 5 6 7 0 1 2 3
5	5 6 7 0 1 2 3 4
6	6 7 0 1 2 3 4 5
7	7 0 1 2 3 4 5 6

Example NB- CCSK mapped on 8-PSK

- A permutation $\prod = [0, 1, 6, 7, 4, 5, 2, 3]$
- Squared Euclidian distance between G_0 and G_1
- $d(G_0, G_1)^2 = \|C(1) - C(0)\|^2 + \|C(2) - C(1)\|^2$
- $d(G_0, G_1)^2 = 4$
- $D(G) = \min_{a \neq b}\{d(G_a, G_b)^2\} = 4$

b	G_b
0	0 1 6 7 4 5 2 3
1	1 6 7 4 5 2 3 0
2	6 7 4 5 2 3 0 1
3	7 4 5 2 3 0 1 6
4	4 5 2 3 0 1 6 7
5	5 2 3 0 1 6 7 4
6	2 3 0 1 6 7 4 5
7	3 0 1 6 7 4 5 2

The sequence order greatly impact the minimum distance of the "code"



Lab-STICC

cnrs

Distance and correlation metrics

$$d(\mathbf{G}_a^p, \mathbf{G}_b^p)^2 = \sum_{i=0}^{p-1} (\mathbf{G}_a^p(i) - \mathbf{G}_b^p(i))^2$$

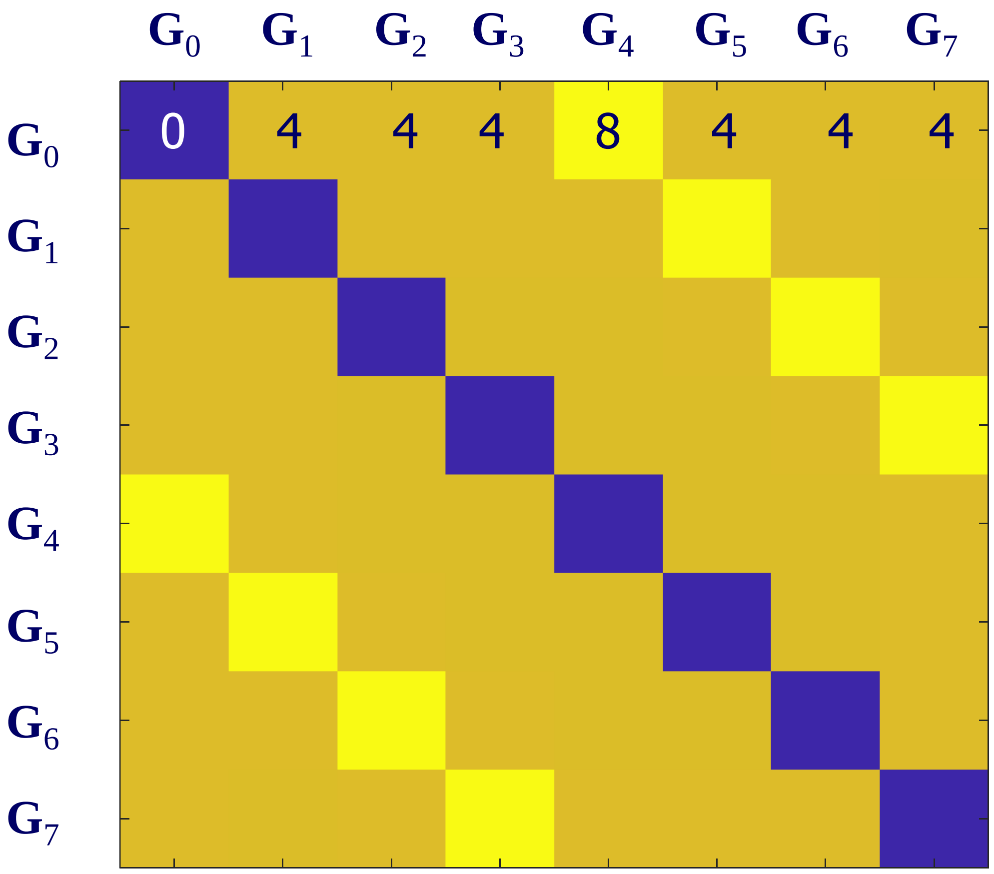
$$= \sum_{i=0}^{p-1} \mathbf{G}_a^p(i)^2 + \mathbf{G}_b^p(i)^2 - 2\mathcal{R}(\mathbf{G}_a^p(i) \times \mathbf{G}_b^p(i)')$$

With a constellation on the unit circle, we have $\mathbf{G}_a^p(i)^2 = 1$ and,

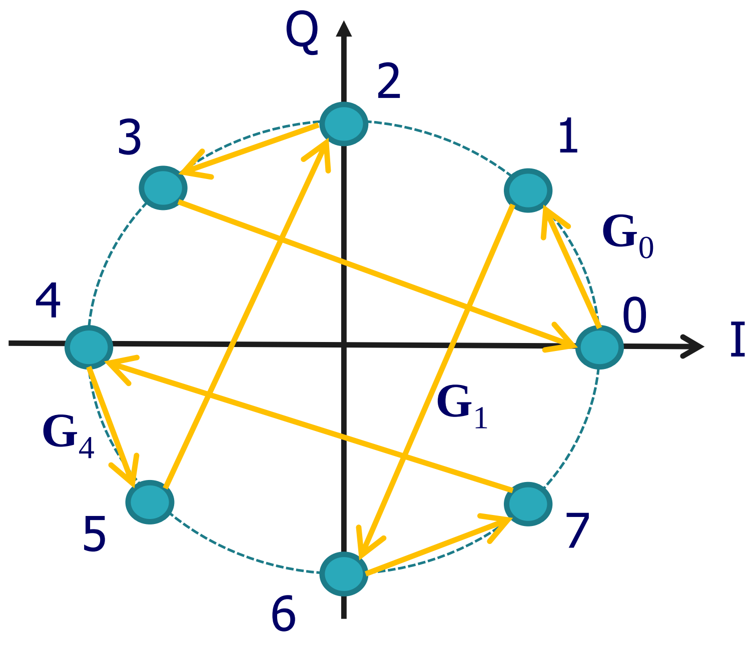
$$d(\mathbf{G}_a^p, \mathbf{G}_b^p)^2 = 2p - 2\mathcal{R}(\langle \mathbf{G}_a^p, \mathbf{G}_b^p \rangle)$$

- Where $< a, b >$ is the inner product of complex sequence.
- Thus $D(G) = \min_{a \neq b}\{d(\mathbf{G}_a, \mathbf{G}_b)^2\} = 2p - 2 \max(\mathrm{Real}(<\mathbf{G}_a, \mathbf{G}_b>, a \neq b))$.

Case p = 2: distance table



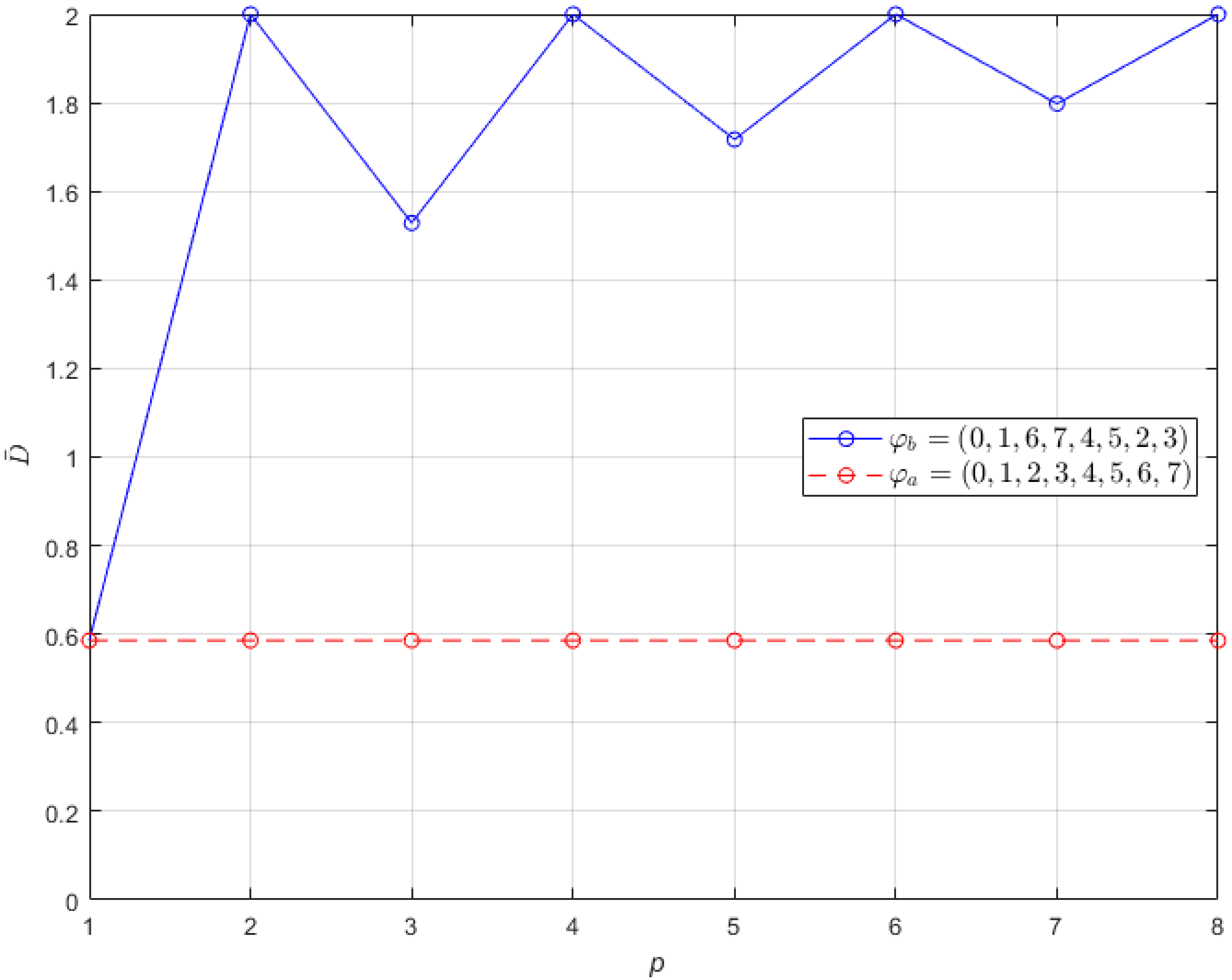
$d(\mathbf{G}_i - \mathbf{G}_j)^2$



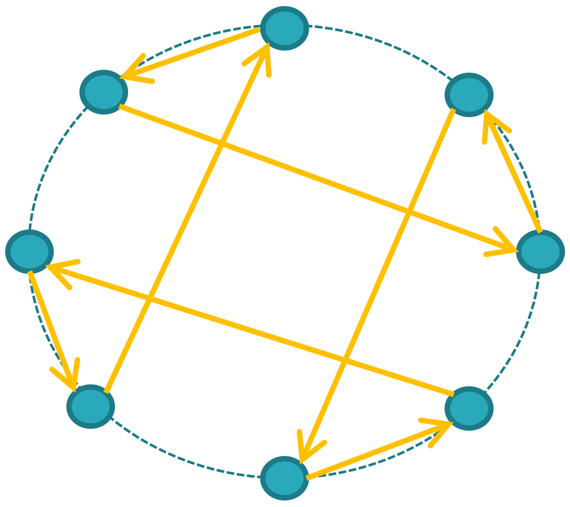
Codewords are either orthogonal or antipodal => bi-orthogonal code

Example: NB- CCSK mapped on 8-PSK

- *Normalized min distance*
  - $\bar{D} = D / p$
- Bi-orthogonal at
  - $p = q / 4$
  - $p = q / 2$
  - $p = 3q / 4$
  - $p = q$

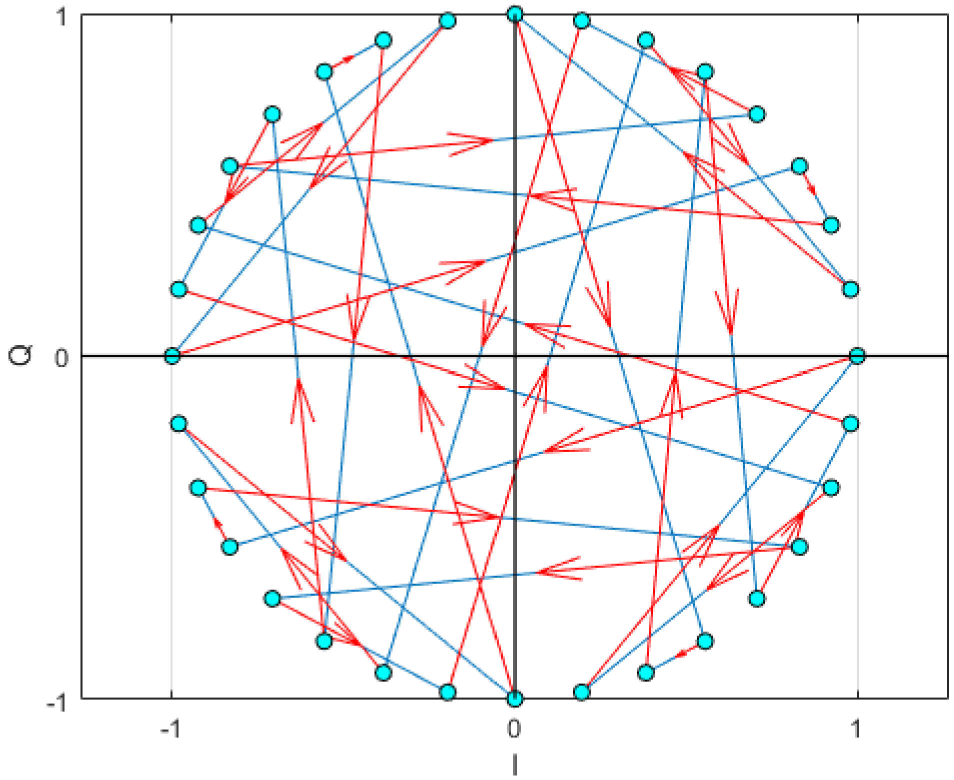


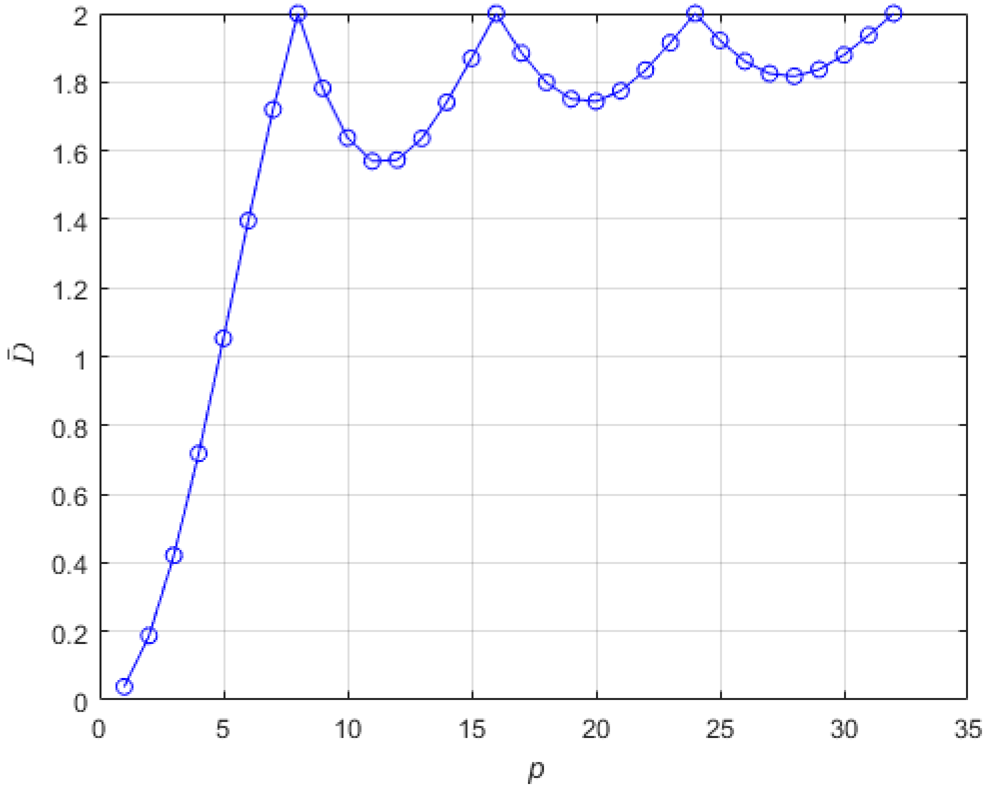
Can we find sequence for GF(64) , GF(256), … ?



- Try all possible permutation is $O((q-1)!)$.
  - permutations for $q = 32$: $2{,}63\text{x}10^{35}$
  - permutations for $q = 64$: $1{,}98\text{x}10^{87}$
  - Permutation for $q = 256$: $3{,}35\text{x}10^{504}$
- From observation of GF(8),
  - 4-fold rotational symmetry
  - Good result at $p = q/4$ implies good results at $p = q/2, 3q/4$ and q
- Constrained symmetric shape and optimization only for $p = q/4$
  - Solution for GF(32)

Example: NB- CCSK mapped on 32-PSK

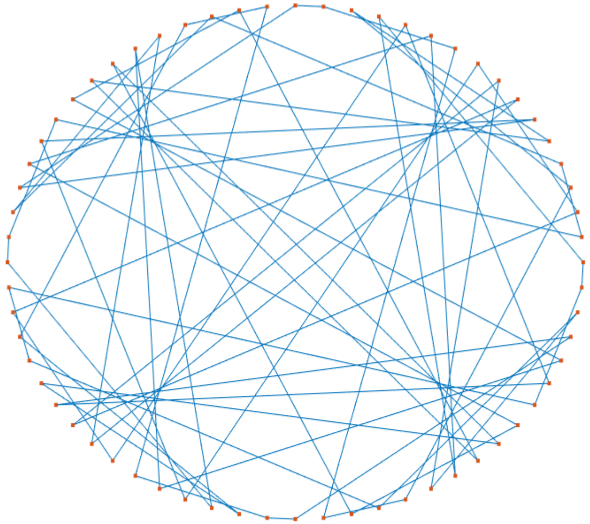




- Optimization on maximizing the minimum distance for *p* =8 using simulated annealing algorithm with 32-PSK .

Research process …

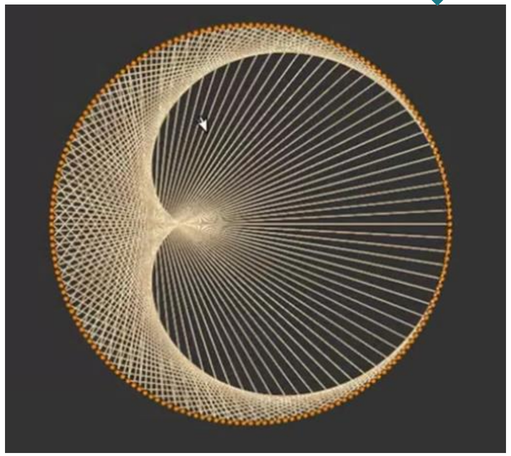
- After some search, we found this nice sequence for $q = 64$



Caustic of a circle

→ Coffee break

Cardioid → n-epicycloid

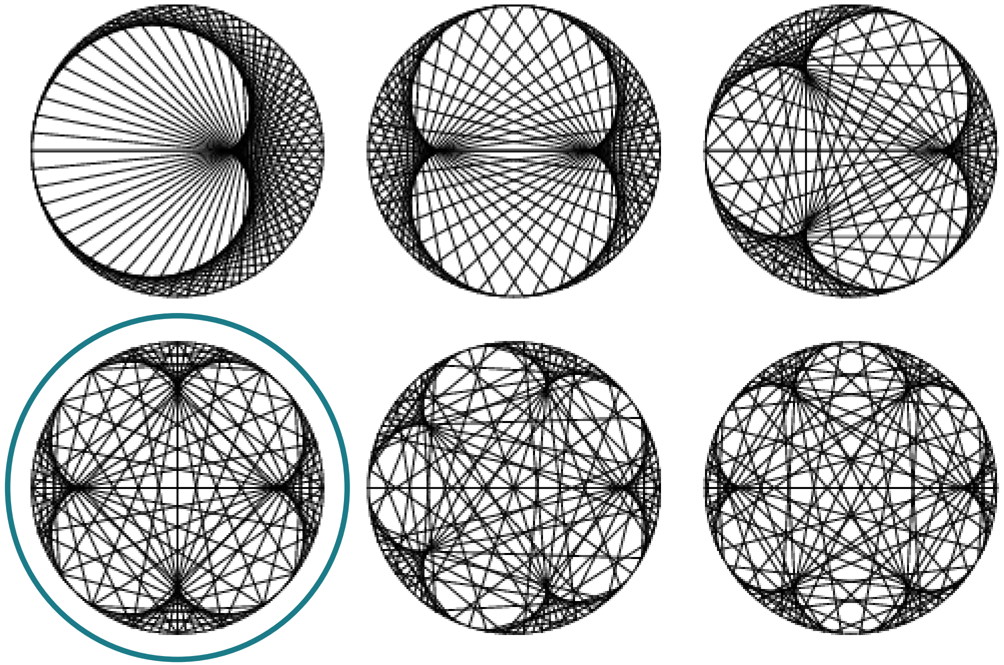


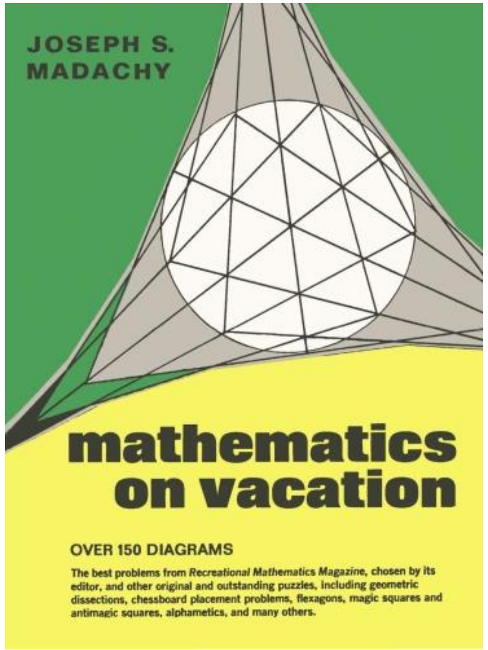
String art mandala

→ Generic construction of a pattern

n-epicycloid

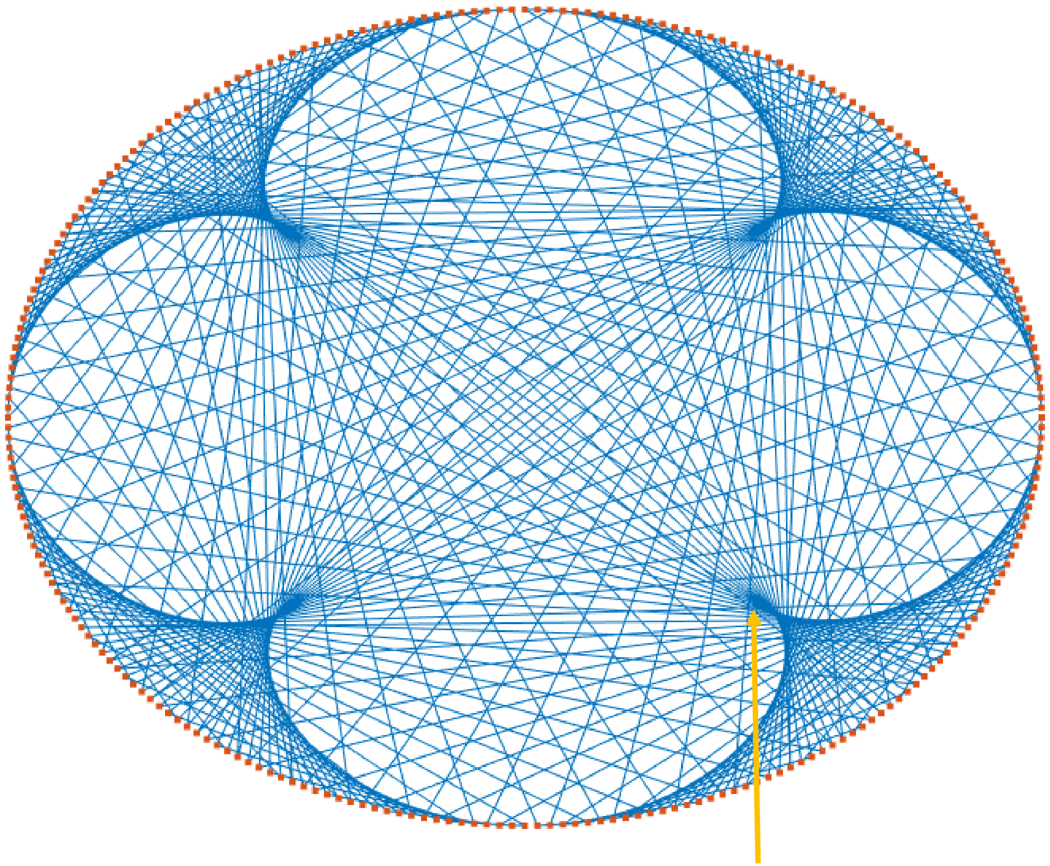
- *n*-epicycloids can also be constructed by beginning with the Diameter of a Circle, offsetting one end by a series of steps while at the same time offsetting the other end by steps *n* times as large. After traveling around the Circle once, an *n*-cusped epicycloid is produced [6].



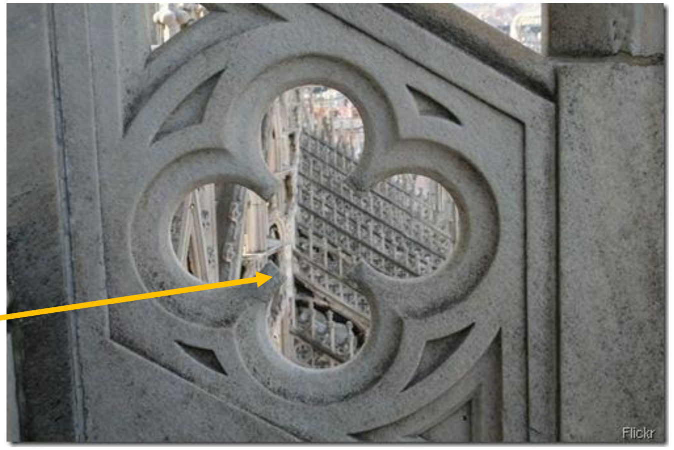


[3] Joseph S. Madachy ‘Mathematics on vacation’,Charles Scribner’s Sons, p219-224, 1966

4 cusps epicycloid



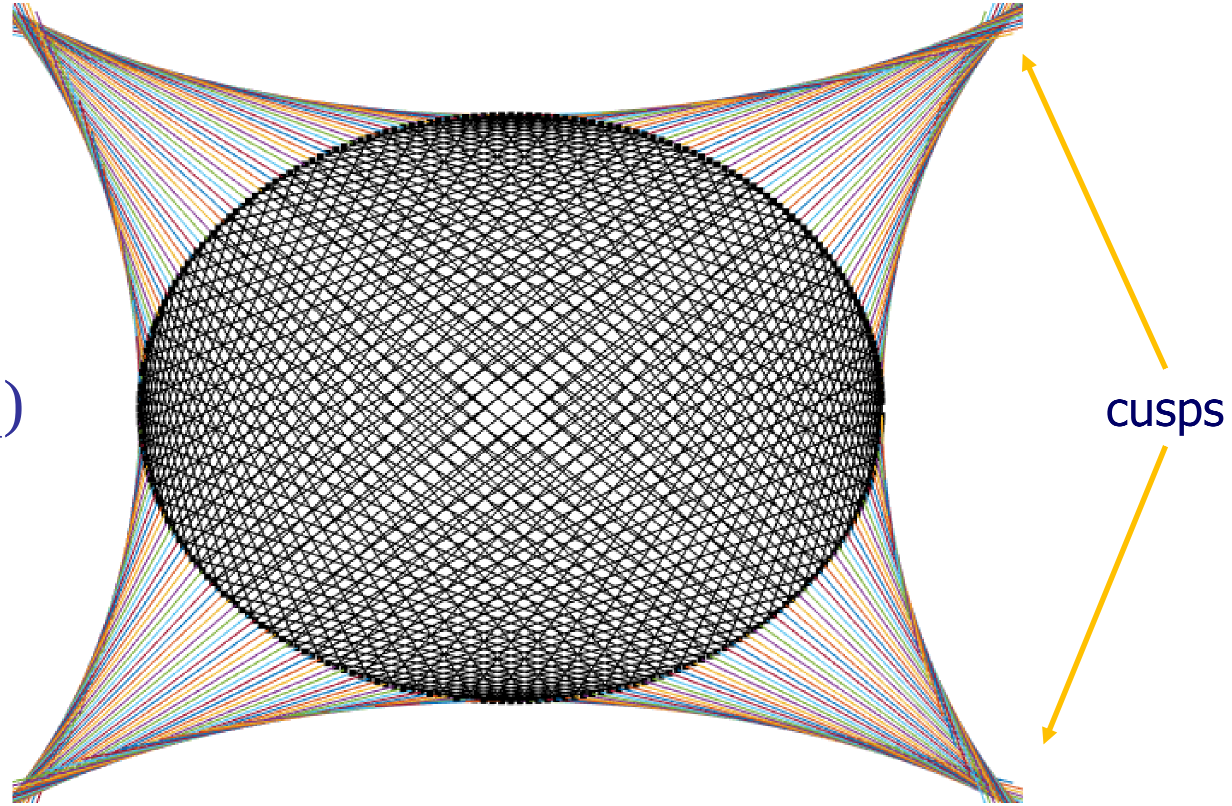
- Sequence construction
  - $\mathbf{G}_0(0) = 0$
  - $\mathbf{G}_0(i+1) = \mathrm{mod}(\mathbf{G}_0(i)*(\mathrm{cusps}+1) +1 , q)$
- Bi-orthogonal at
  - $p = q / 4$
  - $p = q / 2$
  - $p = 3q / 4$
  - $p = q$



Cusp: a pointed end or part where two curves meet

4 cusps hypocycloid or « astroid »

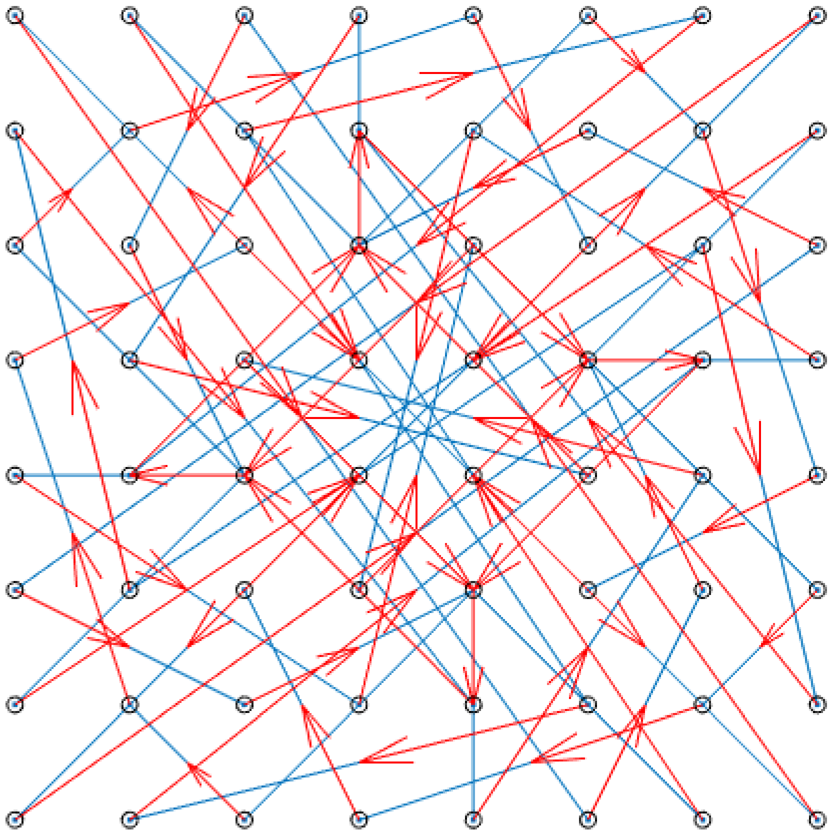
- From the same book [3]
  - **G_0**$(0)=0$
  - **G_0**$(i+1)=\mathrm{mod}(-$ **G_0**$(i)*(\mathrm{cusps}+1), q)$
- Same bi-orthogonal properties



[3] Joseph S. Madachy 'Mathematics on vacation',Charles Scribner's Sons, p219-224, 1966

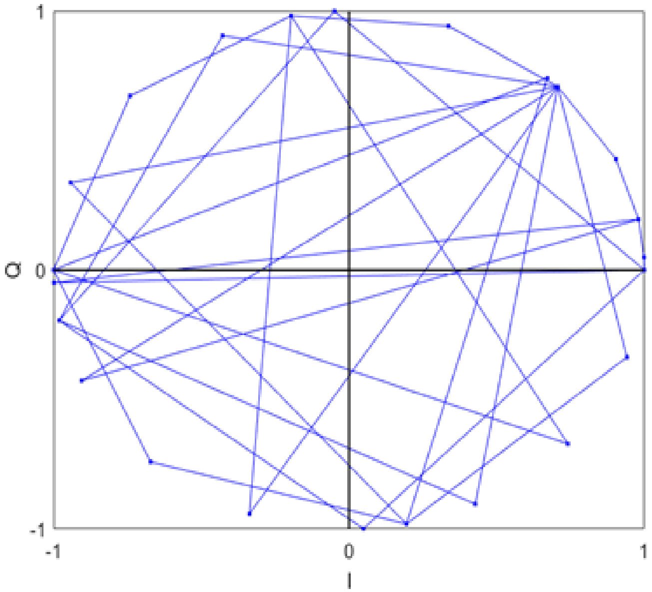
CCSK sequence for 64-QAM constellation

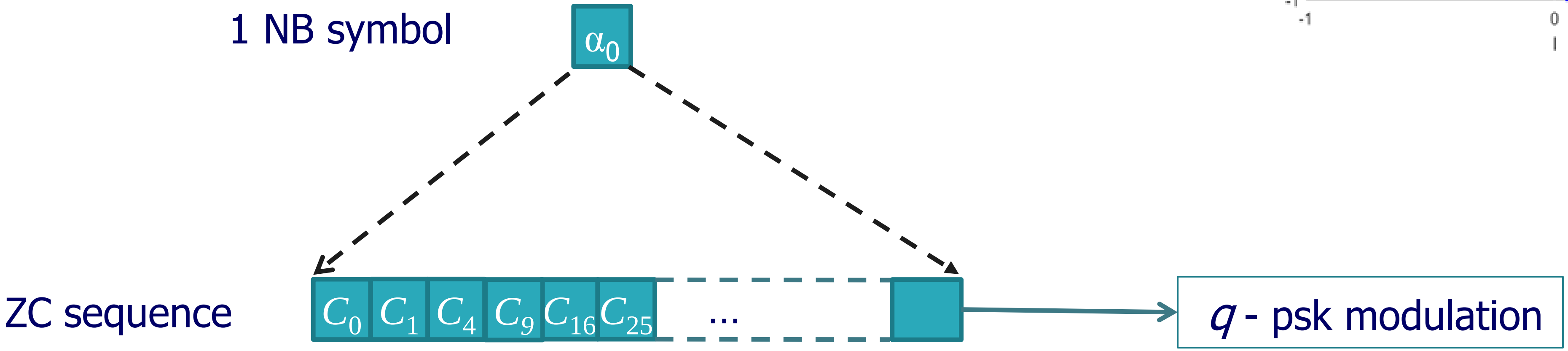
- Use of a QAM constellation
  - Better min distance for $p = 1$
- Permutation Optimization
  - Multi-objective simulated annealing
    - $p = 2$
    - $p = q/4$
  - Rotation symmetry
- Nearly bi-orthogonal



NB-CCSK: Zadoff-Chu sequence

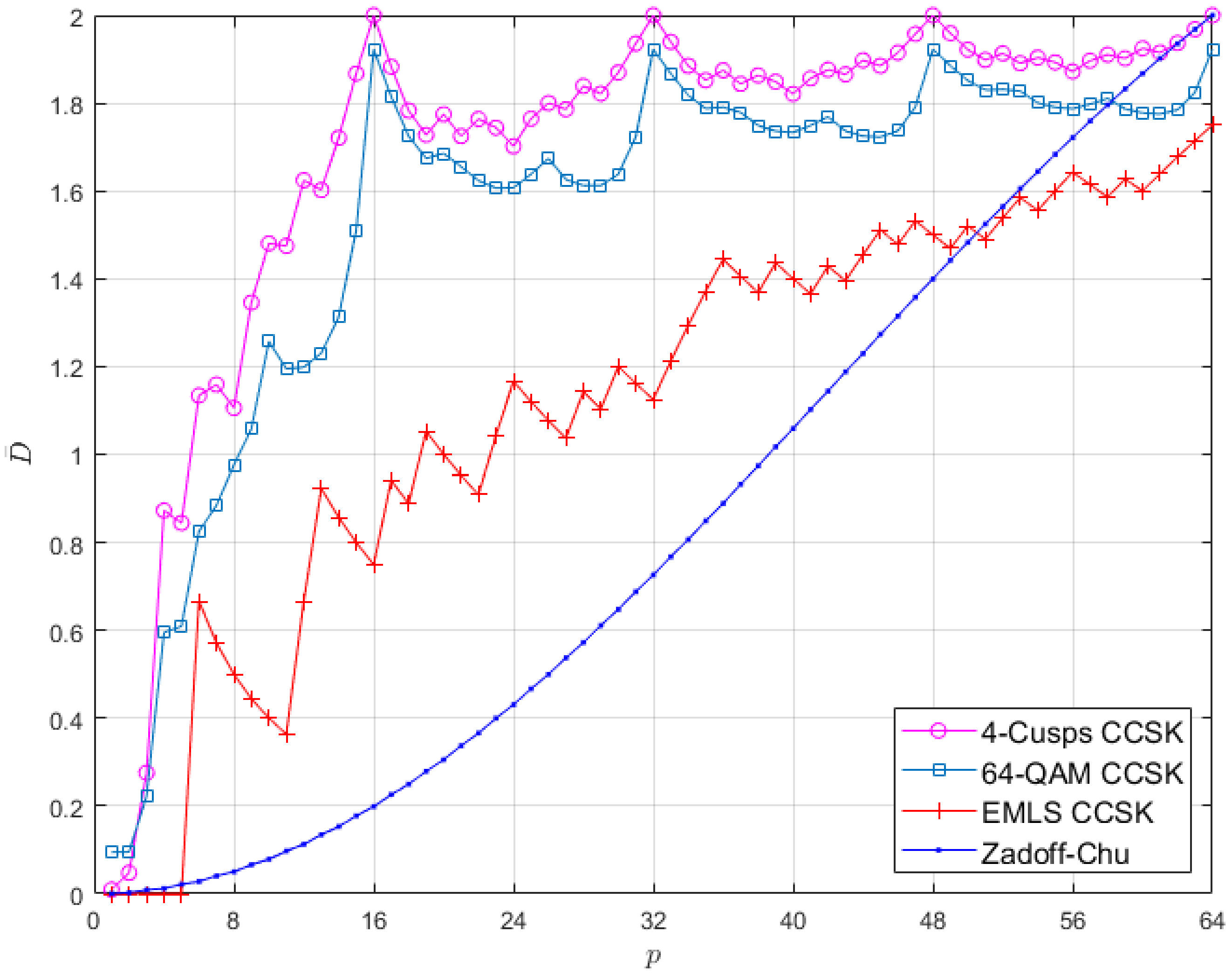
- Zadoff-Chu (ZC) sequence
  - ◇ $C(i) = e^{\,j(2\pi \mathbf{G}_0(i)/q)}$
  - ◇ $\mathbf{G}_0(i) = \mathrm{mod}(i^2, q)$
- Can be considered as a NB-CCSK sequence
- When cyclically shifted, the resulting ZC sequences are uncorrelated with one another.





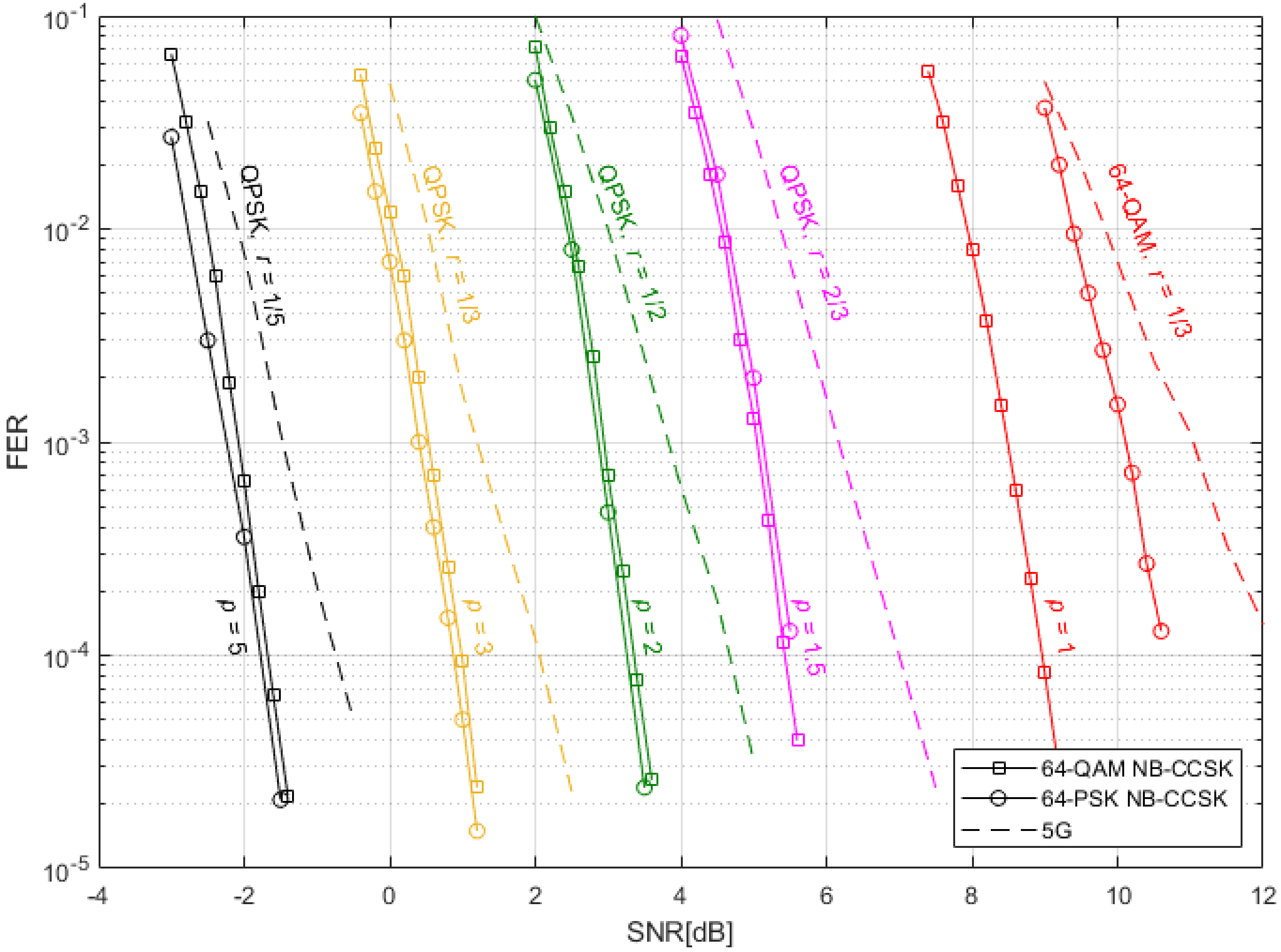
Minimum distance comparison

- EMLS CCSK
  - Binary CCSK
  - Extented Maximum Length Sequence
- For $p = 64$
  - D(Zadoff-Chu) = D(4-Cusp)
- For $2 < p \leq 64$
  - D(4-Cusp)>D(QAM)
- For $p = 1$
  - 64-QAM CCSK outperform the 4-Cusp CCSK



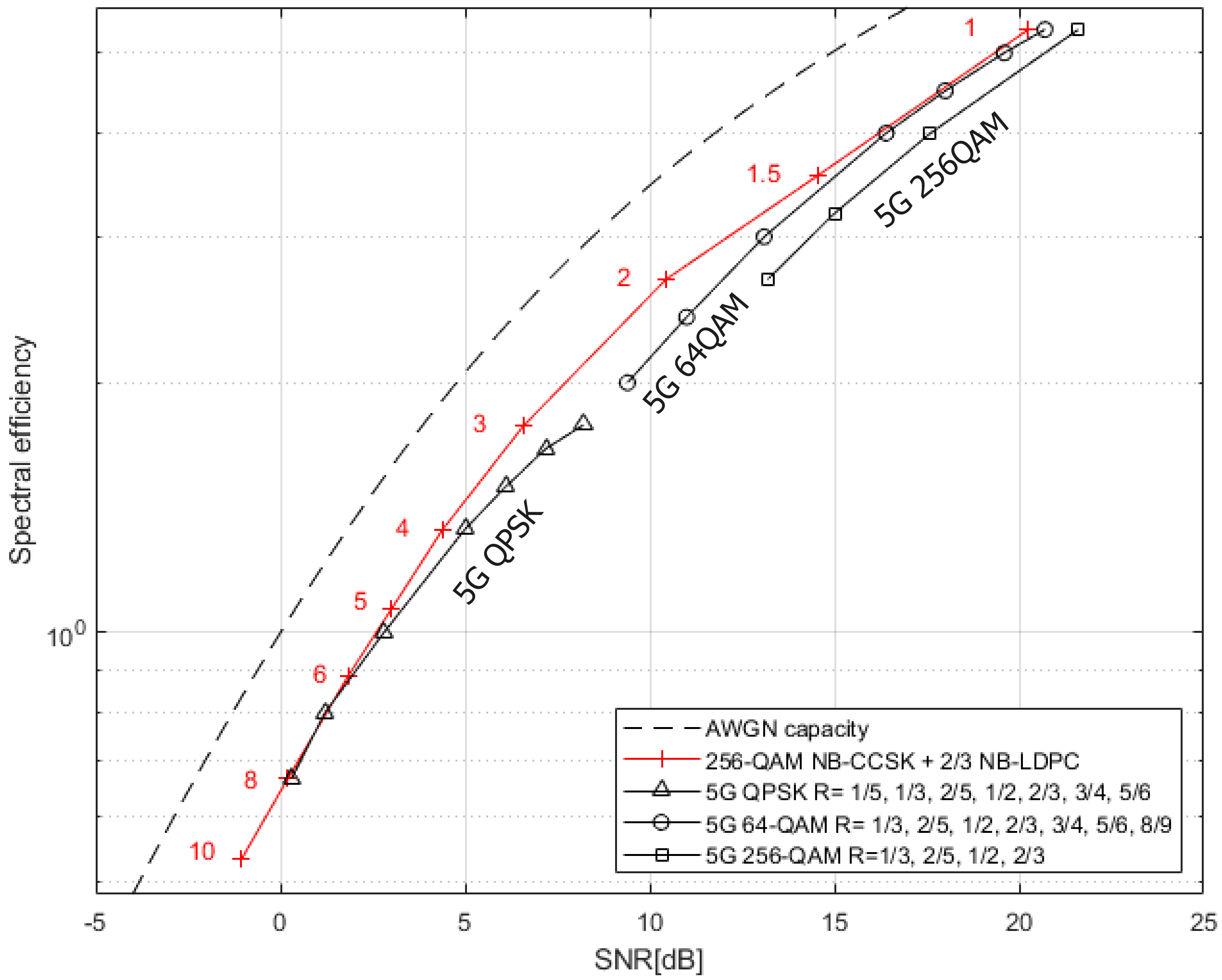
FER of 64-QAM and 64-PSK TCCSK

- NB-LDPC
  - k=20 symbols (120 bits)
  - r_o=1/3
  - n_m=20, n_{op}=25
- 5G LDPC decoder
  - Graph1
  - k=120 bits



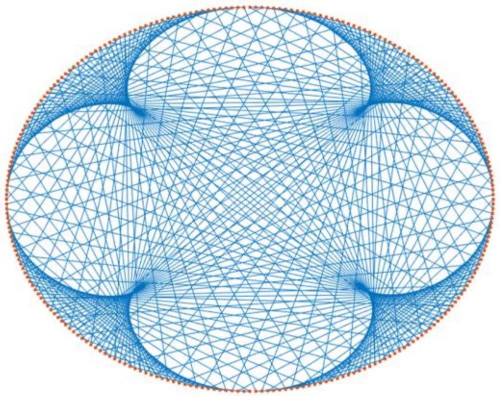
Spectral efficiency of NB-CCSK with 256-QAM constellation

- NB-LDPC
  - ◇ k = 15 symbols (120 bits)
  - ◇ r_o = 2/3
  - ◇ n_m= 70, n_{op} = 80
- 5G LDPC
  - ◇ Graph1
  - ◇ k = 120 bits
- For p=256 (not shown)
  - ◇ Spectral efficiency = 0.02 bits/s/Hz
  - ◇ SNR CCSK=-15.5 dB
  - ◇ SNR Shannon limit = -18.3 dB



Conclusion

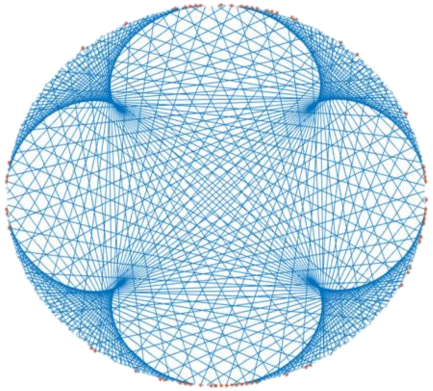
- Rate-adaptive cyclic complex spreading sequence
  - 4 Cusps family
  - Bi-orthogonal at $p = q, 3q/4, q/2, q/4$
  - Rate adaptive inner code for NB-Decoder
    - Good FER performance
  - Wide range of code rates down to very low SNR

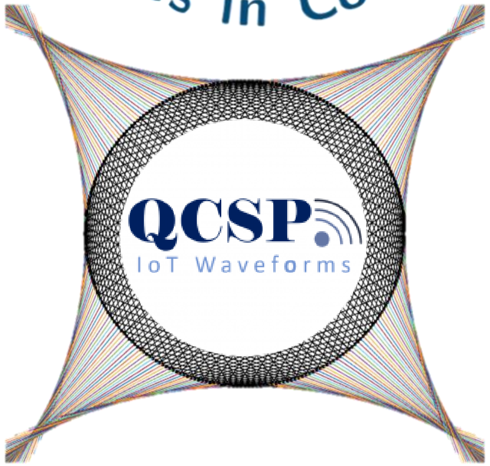


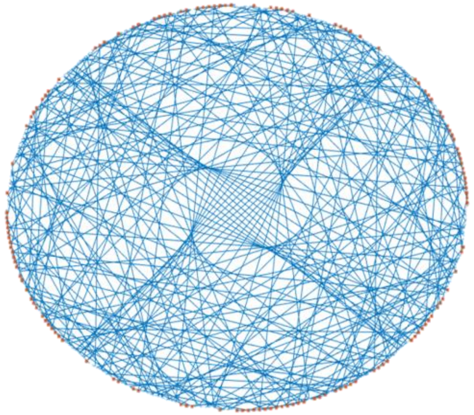
- Extension of this work
  - Tomorrow Special Session on Short Codes and Their Applications
  - E. Boutillon "C4-Sequences: Rate Adaptive Coded Modulation for Few Bits Message"

Rate-Adaptive Cyclic Complex Spreading Sequence for Non-Binary Decoders

- Thank you for your attention







- Questions & Answers

Lab-STICC

cnrs