ANR, Appel à Projets Générique (AAPG 2019)

QCSP Project (ANR-19-CE25-0013-02)

Deliverable D1.1
Specification of Code Parameters and Performance Requirements

Editor:	Valentin Savin (CEA-LETI)
Deliverable nature:	Internal (scope: Consortium and ANR)
Due date:	October 31, 2019
Delivery date:	November 30, 2019
Version:	1.0
Total number of pages:	16 pages
Keywords:	CCSK modulation, achievable rates

Abstract
This deliverable reports on a common set of parameters and requirements to be used for comparing and validating the different solutions developed in WP1. In particular, this concerns the uncoded block size, the native coding rate of the non-binary code, the effective coding rate after cyclic code shift keying modulation, and the operating signal-to-noise ratio. The achievable rates in both the asymptotic and the finite blocklength regime are also reported.

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Executive Summary

Work Package 1 (WP1) of the QCSP project is concerned with the design and the optimization of low coding rate solutions, through combining non-binary (NB) coding, and cyclic code shift keying (CCSK) modulation. Three families of NB codes will be investigated: NB-Turbo code, NB-Polar code, and NB-LDPC code.

The goal of **Task 1.1** is to define a common set of parameters and requirements, such as to enable the evaluation and the comparison of the different solutions investigated in WP1. In particular, this concerns the uncoded block size, the native coding rate of the NB code, the effective coding rate after CCSK modulation.

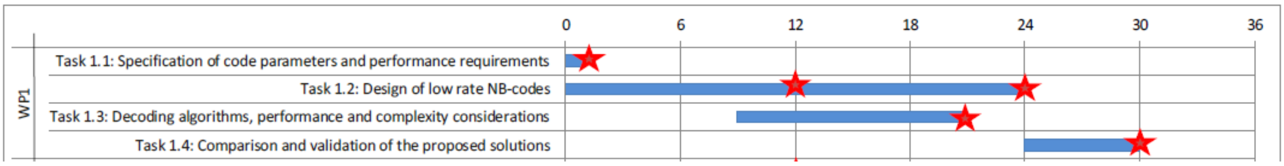


Figure 1: Gantt diagram of WP1

This deliverable is organized as follows:

Section 1 introduces the system model, and derives the achievable coding rates in both the asymptotic and the finite blocklength regimes.

Section 2 defines a common set of parameters and requirements, to be used for the evaluation and the comparison of the different solutions investigated in WP1.

1 System Model and Achievable Rates

1.1 System Model

1.1.1 CCSK Modulation

Throughout this section, we denote by $\mathbb{Z}_q \triangleq \{0, 1, \ldots, q-1\}$ the set of integers comprised between 0 and $q-1$, where $q = 2^p$ is a power of 2. We shall further identify $\mathbb{Z}_q \cong \mathbb{Z}_2^p \triangleq \{0, 1\}^p$, by identifying an integer to its binary representation, $k \in \mathbb{Z}_q \cong (k(0), \ldots, k(p-1)) \in \mathbb{Z}_2^p$.

The Cyclic Code Shift Keying (CCSK) modulation uses a pseudo-random noise (PN) sequence

$$\mathbf{P}_0 = (\mathbf{P}_0(0), \mathbf{P}_0(1), \ldots, \mathbf{P}_0(q-1)), \tag{1.1}$$

which will be written in short as $\mathbf{P}_0 = (\mathbf{P}_0(i))_{i=0,\ldots,q-1}$, where $\mathbf{P}_0(i) \in \{-1, +1\}, \forall i = 0, \ldots, q-1$.

The CCSK modulation maps an element $k \in \mathbb{Z}_q$ to the sequence $\mathbf{P}_k$, defined as the circular left shift of $\mathbf{P}_0$ by k positions, that is $\mathbf{P}_k = (\mathbf{P}_0(i + k \mod q))_{i=0,\ldots,q-1}$.

Throughout this deliverable, integers p and q will be referred to as the *dimension* and *length* of the CCSK modulation.

1.1.2 PN Sequences

An infinite-length pseudo-random sequence $\mathbf{P}$ can be generated by using a Linear Feedback Shift Register (LFSR), as illustrated in Fig. 1.1, where:

- $g_1, g_2, \ldots, g_{p-1}$ are the coefficients of the *feedback polynomial*

$$g(x) = \sum_{i=0}^{p} g_i x^i, \quad \text{with } g_0 = g_p = 1 \tag{1.2}$$

- $(s_{p-1}, \ldots, s_1, s_0)$ is the *state* of the shift register, initialized as $(0, \ldots, 0, 1)$.
- $\oplus$ operations represent the exclusive OR (XOR) gate.
- BPSK denotes the binary phase shift keying modulation of the binary output sequence $(0 \mapsto +1,\ 1 \mapsto -1)$.

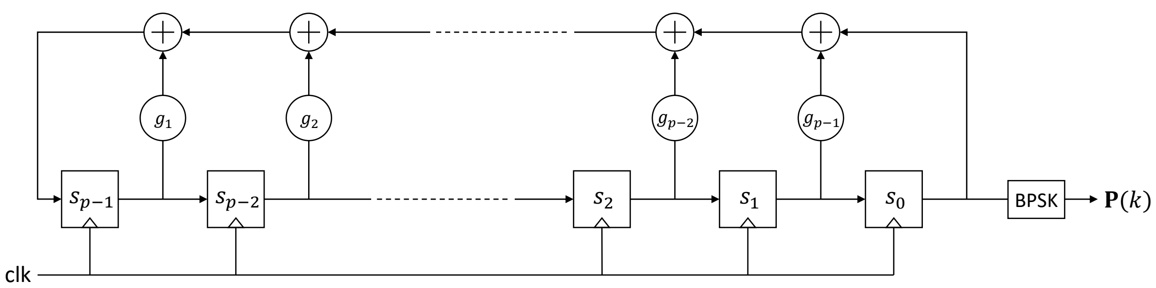


Figure 1.1: LFSR sequence (Fibonacci representation)

The LFSR is maximum-length, *i.e.*, its state cycles through all possible $q-1$ states (except the all-zero state), if and only if the feedback polynomial is primitive. In this case, the generated sequence is referred to as an *m-sequence*. An m-sequence is periodic, of period $q-1$, that is, $\mathbf{P}(i) = \mathbf{P}(i+q-1), \forall i \geq$

0. In particular, $\mathbf{P}(0) = \mathbf{P}(q-1)$. Moreover, the cyclic cross-correlation function takes on only two values, as follows:

$$\theta(k) \triangleq \sum_{i=0}^{q-2} \mathbf{P}(i)\mathbf{P}(k+i) = \begin{cases} q-1, & \text{if } k=0 \\ -1, & \text{if } k=1,\ldots,q-2 \end{cases} \tag{1.3}$$

Going back to the CCSK modulation, we take the PN sequence $\mathbf{P}_0$ from (1.1) to be defined as

$$\mathbf{P}_0 = (\mathbf{P}(0), \mathbf{P}(1), \ldots, \mathbf{P}(q-2), \mathbf{P}(q-1)), \tag{1.4}$$

where $\mathbf{P}$ is an m-sequence. Since the length of $\mathbf{P}_0$ is q, which is 1 greater than the period of $\mathbf{P}$, the cross-correlation function of $\mathbf{P}_0$ is different from the cross-correlation function of $\mathbf{P}$. Thus, defining

$$\theta_0(k) \triangleq \sum_{i=0}^{q-1} \mathbf{P}_0(i)\mathbf{P}_0(k+i \mod q), \quad \forall k = 0, \ldots, q-1, \tag{1.5}$$

we have $\theta_0(0) = q$, but for $k \neq 0$, the absolute value $|\theta_0(k)|$ may exceed 1 (although its value is still small with respect to q).

The feedback polynomials used throughout this deliverable for various p values, together with the minimum, maximum and average $|\theta_0(k)|$ value, for $k \neq 0$, are given in Table 1.1.

Table 1.1: Feedback polynomials for various p values

p	Feedback polynomial $g(x)$	$\min_{k\neq 0} \lvert\theta_0(k)\rvert$	$\max_{k\neq 0} \lvert\theta_0(k)\rvert$	$\text{ave}_{k\neq 0} \lvert\theta_0(k)\rvert$
$p=6$	$g(x) = x^6 + x^5 + x^4 + x + 1$	0	12	2.48
$p=7$	$g(x) = x^7 + x^3 + 1$	0	20	4.94
$p=8$	$g(x) = x^8 + x^4 + x^3 + x^2 + 1$	0	25	6.95
$p=9$	$g(x) = x^9 + x^4 + 1$	0	32	10.07
$p=10$	$g(x) = x^{10} + x^3 + 1$	0	56	13.32
$p=11$	$g(x) = x^{11} + x^2 + 1$	0	76	19.85
$p=12$	$g(x) = x^{12} + x^9 + x^7 + x^6 + 1$	0	132	28.16

Remark [Alternative CCSK mapping] Let $\mathbf{P}_0 = (\mathbf{P}(0), \mathbf{P}(1), \ldots, \mathbf{P}(q-2))$ be an m-sequence of length $q-1$, and $\mathbf{0}$ denote the all-zero sequence of length $q-1$ (or the all $+1$ sequence, assuming that the m-sequence $\mathbf{P}_0$ is BPSK modulated, as in Fig 1.1). One may define an alternative CCSK modulation, of length $q-1$, by mapping $k=0$ to the sequence $\mathbf{0}$, and $k = 1, \ldots, q-1$ to the sequence $\mathbf{P}_{k-1}$, defined as the circular left shift of $\mathbf{P}_0$ by $k-1$ positions. It is easily seen that in this case $\theta_0(0) = q-1$, and $\theta_0(k) = -1$, $\forall k = 1, \ldots, q-1$, thus preserving the cross-correlation properties of the m-sequence. But the main advantage of such a mapping is when the CCSK modulation is combined with non-binary coding over the Galois field of size q. Indeed, let $\text{GF}(q) \triangleq \{0, \alpha^0, \alpha^1, \ldots, \alpha^{q-2}\}$ denote the Galois field of size q, where α is a primitive element. Using the above mapping, GF element 0 is mapped to the sequence $\mathbf{0}$, while a GF element α^k, $k = 0, \ldots, q-2$, is mapped to the sequence $\mathbf{P}_k$. Hence, multiplying a sequence of GF symbols by a non-zero element α^k is equivalent to circularly shifting all the mapped CCSK symbols by k positions to the left. In other words, the multiplicative structure of the Galois field matches the cyclic rotations of the CCSK sequence, which may be particularly useful in case of inaccurate chip synchronization. We shall refer to such a CCSK mapping as being *GF compliant*, however, **through the rest of the deliverable we shall consider the length-q CCSK mapping as defined in Section 1.1.1**.

1.1.3 Channel Model

We shall assume that the CCSK modulated signal undergoes **real additive white Gaussian noise**, with variance σ^2. The following notation will be used in the subsequent sections.

- U denotes a uniform random variable, with values in $\mathbb{Z}_q$. Realizations of a U represent unmodulated symbols (input to the CCSK modulation). We shall also write $U = (U(0), \ldots, U(p-1))$, where $U(i)$ are binary random variables, defined by the binary representation of U.
- $X = (X(0), X(1), \ldots, X(q-1))$ denotes the random variable defined by modulating U. Hence, $X = \mathbf{P}_k \Leftrightarrow U = k$.
- $Y = (Y(0), Y(1), \ldots, Y(q-1))$ denotes the received signal. Hence,

$$Y(k) = X(k) + Z(k), \tag{1.6}$$

  where $Z(k)$ are real-valued, mutually independent, normal random variables, with mean 0 and variance σ^2.

Throughout this deliverable, the Signal-to-Noise Ratio value is defined as $\text{SNR} \triangleq -10\log_{10}(\sigma^2)$ (dB).

1.1.4 Demodulation

Symbol-Level Demodulation

Given the received signal Y, the symbol-level Log-Likelihood Ratio (LLR) values are defined by

$$\Gamma(k) \triangleq \log \frac{\Pr(U = 0 \mid Y)}{\Pr(U = k \mid Y)}, \quad \forall k \in \mathbb{Z}_q \tag{1.7}$$

Hence, we have

$$\Gamma(k) = \log \frac{\Pr(U = 0 \mid Y)}{\Pr(U = k \mid Y)} = \log \frac{\Pr(X = \mathbf{P}_0 \mid Y)}{\Pr(X = \mathbf{P}_k \mid Y)} = \sum_{i=0}^{q-1} \log \frac{\Pr(X(i) = \mathbf{P}_0(i) \mid Y(i))}{\Pr(X(i) = \mathbf{P}_k(i) \mid Y(i))} = \tag{1.8}$$

$$= \sum_{i=0}^{q-1} \frac{\mathbf{P}_0(i) - \mathbf{P}_k(i)}{2} \log \frac{\Pr(X(i) = +1 \mid Y(i))}{\Pr(X(i) = -1 \mid Y(i))} \tag{1.9}$$

Assuming the AWGN channel model from the previous section, we have $\log \frac{\Pr(X(i)=+1 \mid Y(i))}{\Pr(X(i)=-1 \mid Y(i))} = \frac{2}{\sigma^2} Y(i)$, and therefore

$$\Gamma(k) = \frac{1}{\sigma^2} \left(\sum_{i=0}^{q-1} \mathbf{P}_0(i) Y(i) - \sum_{i=0}^{q-1} \mathbf{P}_k(i) Y(i) \right) \tag{1.10}$$

$$= \frac{1}{\sigma^2} \left(Y \cdot \mathbf{P}_0 - Y \cdot \mathbf{P}_k \right) \tag{1.11}$$

where $Y \cdot \mathbf{P} \triangleq \sum_i Y(i)\mathbf{P}(i)$ denotes the usual dot product of sequences (vectors) Y and $\mathbf{P}$.

Since that $\mathbf{P}_k$ is a circular shifted version of $\mathbf{P}_0$, dot products $Y \cdot \mathbf{P}_k$, $k = 0, \ldots, q-1$, can be conveniently computed by using the Discrete Fourier Transform, denoted in the sequel by $\mathcal{F}$. Indeed, let $Y * \mathbf{P}$ denote the circular cross-correlation vector of Y and $\mathbf{P}$, defined by

$$(Y * \mathbf{P})(k) \triangleq \sum_{i=0}^{q-1} Y(i) P(k + i \mod q) \tag{1.12}$$

Hence, $Y \cdot \mathbf{P}_k = (Y * \mathbf{P}_0)(k)$, and therefore we get

$$Y \cdot \mathbf{P}_k = (Y * \mathbf{P}_0)(k) = \mathcal{F}^{-1}\left(\mathcal{F}(Y)^* \cdot \mathcal{F}(\mathbf{P}_0)\right)(k) \tag{1.13}$$

where $\mathcal{F}(Y)^*$ denotes the complex conjugate of $\mathcal{F}(Y)$.

From the above LLR values, the probability distribution of U conditional on the received sequence Y can be computed by

$$\Pi(k) \triangleq \Pr(U = k \mid Y) = \frac{e^{-\Gamma(k)}}{\sum_{k' \in \mathbb{Z}_q} e^{-\Gamma(k')}} \tag{1.14}$$

Bit-Level Demodulation

Bit-level demodulation computes bit-level LLR values, defined by

$$L(i) \triangleq \log \frac{\Pr\left(U(i) = 0 \mid Y\right)}{\Pr\left(U(i) = 1 \mid Y\right)}, \quad \forall i = 0, \ldots, p-1 \tag{1.15}$$

Hence, bit-level LLR values can be computed by marginalizing the probability distribution of U conditional on the received sequence Y, as follows:

$$L(i) = \log \frac{\sum_{k \in \mathbb{Z}_q : k(i) = 0} \Pi(k)}{\sum_{k \in \mathbb{Z}_q : k(i) = 1} \Pi(k)} \tag{1.16}$$

It is worth noticing that the bit-level demodulation is not needed in case that CCSK modulation is combined with non-binary coding over a size-q alphabet.

1.1.5 Coded CCSK Modulation

Assume that unmodulated symbols are encoded by a (binary or non-binary) code C. Precisely, C encodes K *source symbols* into N *coded symbols*, which are (first, converted to $\mathbb{Z}_q$ symbols, if needed, and) then modulated by the CCSK modulation and sent over the AWGN channel. Received signal is first the demodulated, by using either the symbol-level or bit-level demodulation described above, depending on the code type, and then decoded.

We shall distinguish between

- the *(native) coding rate*, defined as $R \triangleq \dfrac{K}{N}$, and
- the *effective coding rate*, defined as $R_{\mathit{eff}} \triangleq \dfrac{p}{q} R$.

To simplify the terminology, simply using *rate* or *coding rate* shall always refer to the native coding rate R.

1.2 Achievable Rates – Asymptotic Blocklength Regime

1.2.1 Non-binary coding

By Shannon's noisy-channel coding theorem [1], the *maximum achievable (native coding) rate*, denoted in the sequel by R, is given by mutual information between the input U of the CCSK modulation and the output Y of the channel

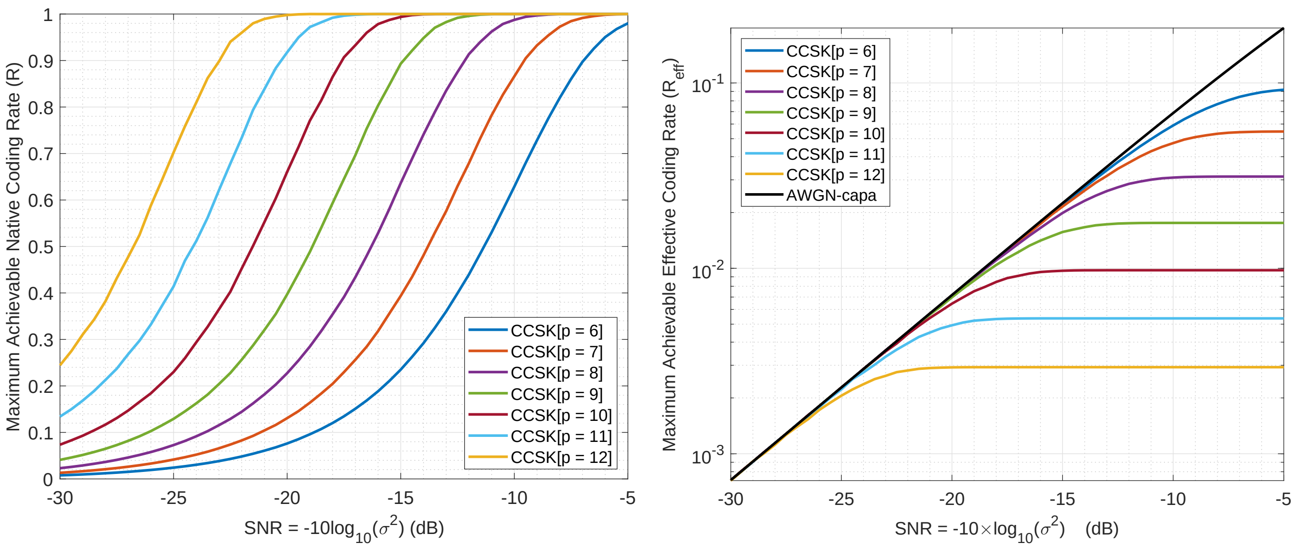
$$R \triangleq I(U;Y) = H(U) - H(U|Y), \tag{1.17}$$

where U denotes the channel input, which is assumed to be uniformly distributed[1] over $\mathbb{Z}_q$, Y denotes the channel output, $H(U)$ denotes the entropy of U, and $H(U|Y)$ denotes the conditional entropy of U given Y. We shall consider both entropy values defined by a base-q logarithm, such that $R \in [0,1]$. Hence, we have $H(U) = 1$. The conditional entropy $H(U|Y)$ can be estimated numerically by Monte-Carlo simulation, by averaging over the channel output Y,

$$H(U|Y) = \mathbb{E}_Y\left[-\sum_{k\in\mathbb{Z}_q} \Pi(k)\log_q \Pi(k)\right], \tag{1.18}$$

where $\Pi(k) \triangleq \Pr(U = k \mid Y)$ denotes the conditional probability distribution of U given Y, as defined in equation (1.14).

Fig. 1.2(a) shows the maximum achievable rate R as a function of the SNR, for $p = 6$ (right-most curve) to $p = 12$ (left-most curve corresponds). For a fair comparison, the overhead introduced by the CCSK modulation itself must also be taken into account. Thus, in Fig. 1.2(b), we plot the maximum achievable *effective* rate $R_{\mathit{eff}} = \frac{p}{q}R$ as a function of the SNR, for $p = 6, \ldots, 12$.



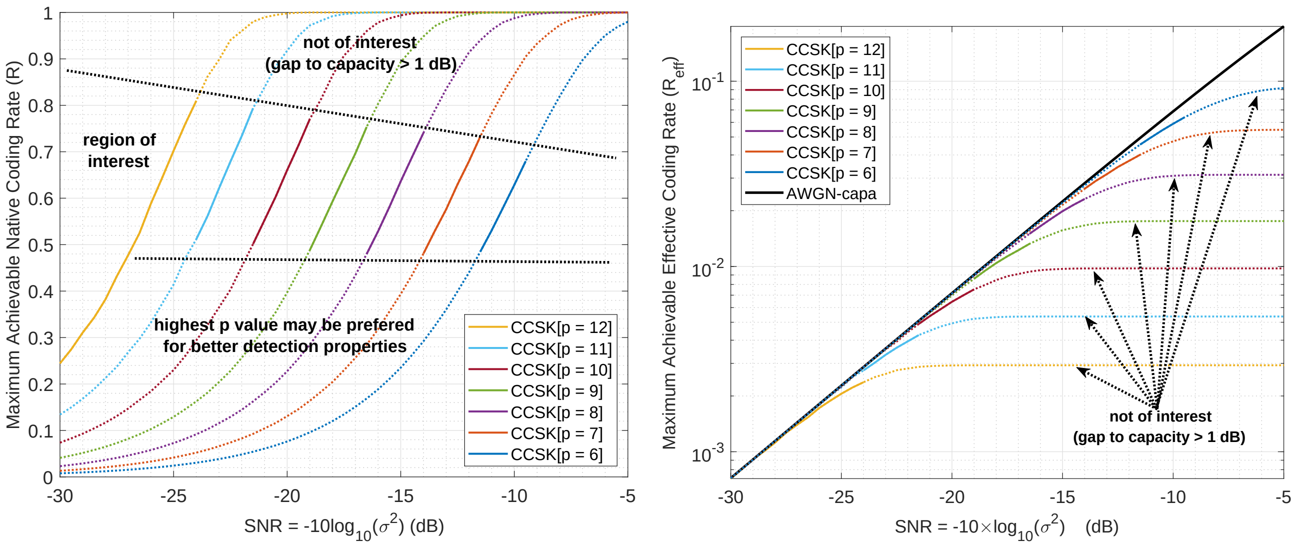
(a) Maximum achievable native coding rates (R)

(b) Maximum achievable effective coding rates (R_{eff})

Figure 1.2: Maximum achievable native and effective coding rates (asymptotic code length)

Fig. 1.3 shows the same plots as Fig. 1.2, but highlights a *region of interest*, delimited to the north by a region where the gap to capacity (in terms of effective coding rate, see Fig. 1.3(b)) is higher than 1 dB, and to the south by a region where several p values may provide a maximum achievable effective coding rate within 1 dB from the capacity, among which we consider the highest p value as being preferred, due to possible better detection and synchonization properties.

[1] Since the channel is symmetric, the channel capacity is achieved for the uniform distribution.



(a) Maximum achievable native coding rates (R)

(b) Maximum achievable effective coding rates (R_{eff})

Figure 1.3: Maximum achievable native and effective coding rates – region of interest (asymptotic code length)

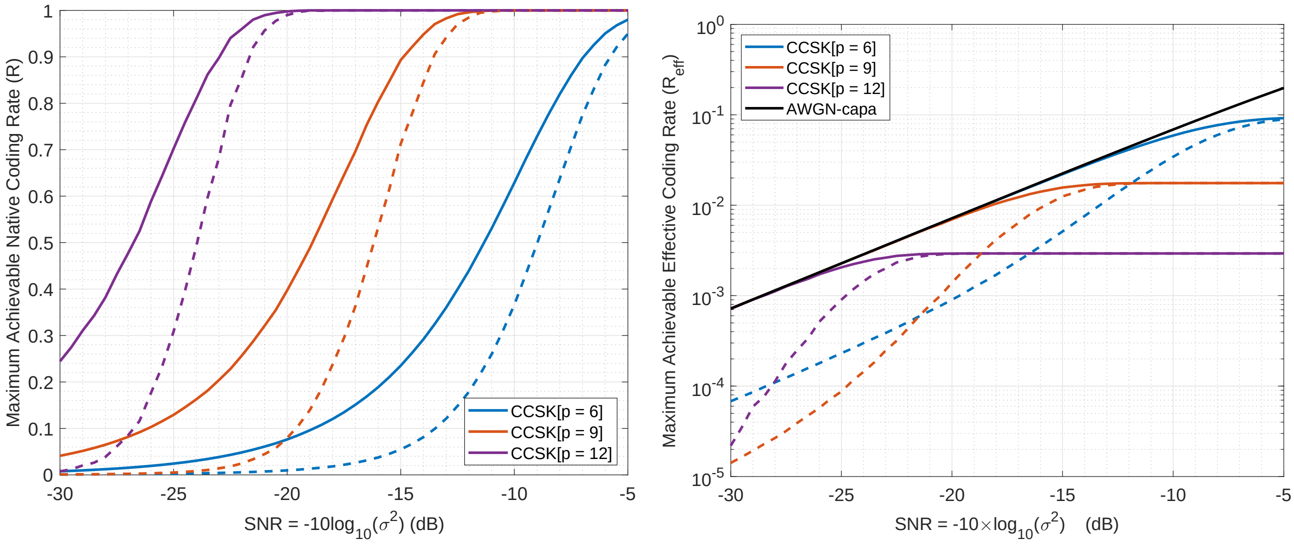
1.2.2 Binary coding

To estimate the advantage of non-binary coding with respect to binary coding, we consider the *bit-level marginalization* of the non-binary channel, consisting of the channels $(U(i), L(i))$, for $i = 0, \ldots, p-1$, with binary input $U(i)$ and output $L(i)$ defined in (1.15). Hence, in case of *binary coding*, the *maximum achievable (native coding) rate* is given by

$$R_{bin} \triangleq \frac{1}{p} \sum_{i=0}^{p-1} I(U(i); L(i)) \tag{1.19}$$

We note that $I(U(i); L(i)) = H(U(i)) - H(U(i) \mid L(i)) = 1 - H(U(i) \mid L(i))$, where the last conditional entropy term can be estimated numerically by Monte-Carlo simulation. Since the bit-level marginalization incurs a loss of mutual information, we have $R_{bin} \leq R$.

Fig. 1.4 shows the maximum achievable rates for non-binary coding (solid curves) vs. binary coding (dashed curves), for $p = 6, 9, 12$. The advantage of non-binary coding is clearly seen from this figure, in terms of both native and effective coding rates.



(a) Maximum achievable native coding rates (R, R_2) (b) Maximum achievable effective coding rates $(R_{\mathit{eff}}, R_{2\ \mathit{eff}})$

Figure 1.4: Maximum achievable native and effective coding rates: non-binary (solid curves) vs. binary (dashed curves) coding (asymptotic code length)

1.3 Achievable Rates – Finite Blocklength Regime

In the non-asymptotic regime, there are no exact formulas for the maximal achievable rate as a function of the code length N, and error probability ε. However, in [2], it was shown that the backoff from channel capacity can be accurately and succinctly characterized by a parameter known as *channel dispersion*. Specifically, the maximum achievable coding rate, denoted by R^*, can be tightly approximated by

$$R^* \approx R - \sqrt{\frac{V}{N}} Q^{-1}(\varepsilon), \tag{1.20}$$

where R is the channel capacity (maximum rate achievable in the asymptotic regime), and V is the channel dispersion. We use the above approximation (known as the *normal approximation*) as a definition of the maximum achievable coding rate in the finite code-length regime. It is worth noticing that here *achievable* makes the implicit assumption of random coding and maximum-likelihood (ML) decoding.

Using [2, Theorem 49, and equations (240), (270)], it can be seen (after some computations) that the channel dispersion parameter can be computed as

$$V = H_2(U \mid Y) - H(U \mid Y)^2, \tag{1.21}$$

where $H(U \mid Y)$ is the conditional entropy of the channel input U given the channel output Y (see Section 1.2.1), and

$$H_2(U \mid Y) \triangleq \mathbb{E}_Y \left[- \sum_{k \in \mathbb{Z}_q} \Pi(k) \log_q^2 \Pi(k) \right] \tag{1.22}$$

where $\Pi(k) \triangleq \Pr(U = k \mid Y)$ denotes the conditional probability distribution of U given Y, as defined in equation (1.14). Hence, $H_2(U \mid Y)$ can be conveniently estimated by Monte-Carlo simulation.

Finally, we note that we are interested in the maximum achievable rate when fixing the information blocklength K, rather than the coded blocklength N. To do so, we replace $R^* = K/N$ in (1.20), and then solve for N in the equation $K/N = R - \sqrt{V/N} Q^{-1}(\varepsilon)$, which gives

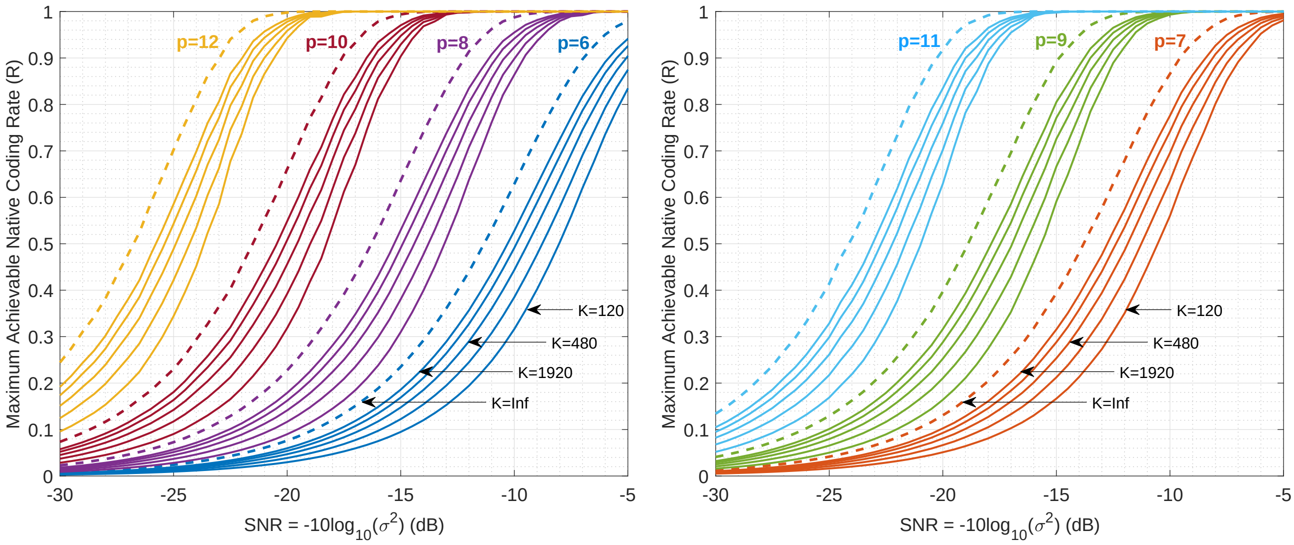
$$N(K, R, V, \varepsilon) = \left(\frac{V_\varepsilon + \sqrt{V_\varepsilon^2 + 4RK}}{2R} \right)^2, \text{ where } V_\varepsilon = \sqrt{V} Q^{-1}(\varepsilon) \tag{1.23}$$

Hence, for an information blocklength K, the maximum achievable rate R^* is given by

$$R^* = \frac{K}{\lceil N(K, R, V, \varepsilon) \rceil} \tag{1.24}$$

where $\lceil N \rceil$ denotes the least integer greater than or equal to N.

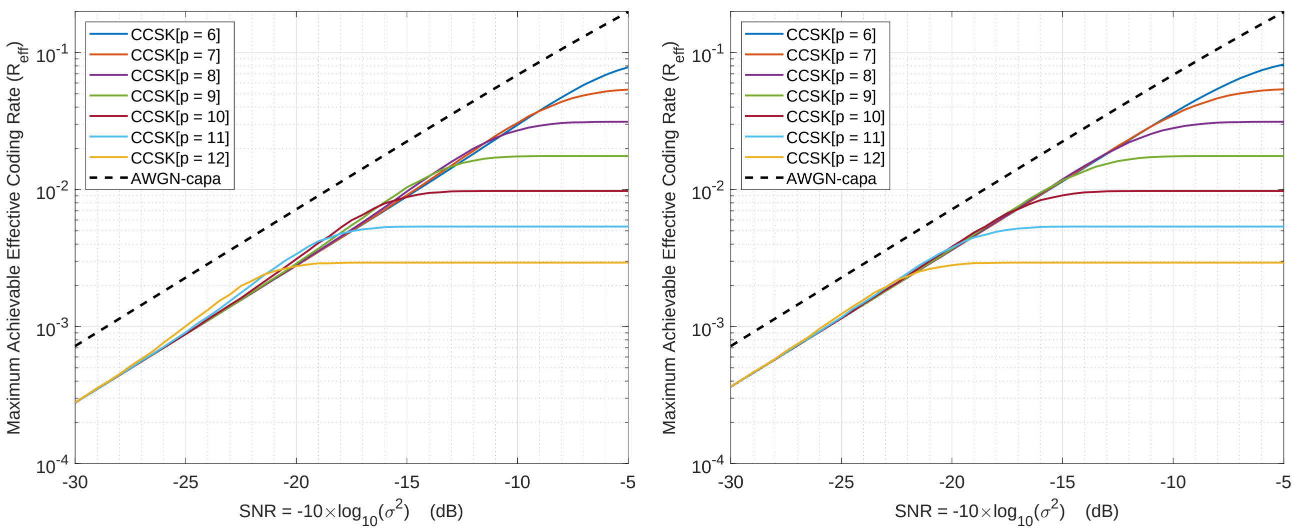
Fig. 1.5 shows the maximum achievable native coding rates, in both the asymptotic and finite blocklength regime. Asymptotic rates are shown as dashed curves, while the finite blocklength regime curves are shown as solid curves. For the finite blocklength regime, we consider an information blocklength $K = 120, 240, 480, 960, 1920$, and a target error probability value $\varepsilon = 10^{-10}$.



(a) $p = 6, 8, 10, 12$ (b) $p = 7, 9, 11$

Figure 1.5: Maximum achievable native coding rates: asymptotic vs. finite ($K = 120, 240, 480, 960, 1920$) blocklengths

Finally, Fig. 1.6 shows the maximum achievable native coding rates in the finite block-length regime, for information blocklength values $K = 120, 240, 480, 960, 1920$. The asymptic capacity of the AWGN channel (black, dashed curve) is also shown, as a reference curve.



(a) $K = 120$ (b) $K = 240$

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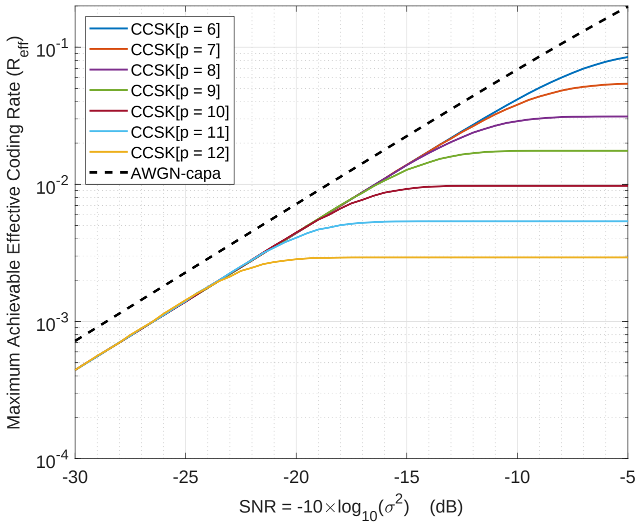
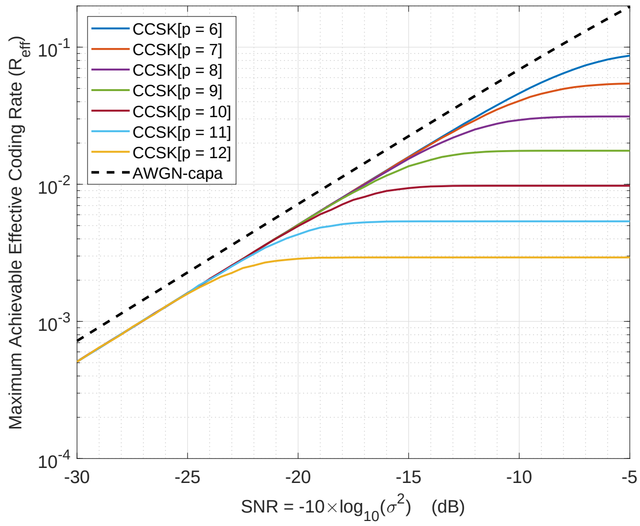
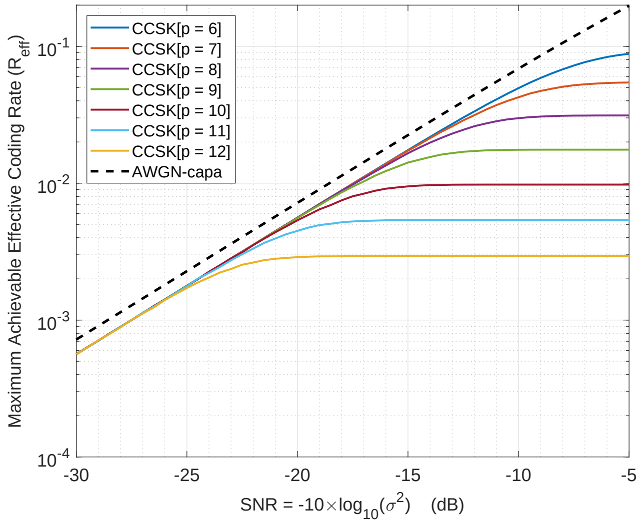


(c) $K = 480$



(d) $K = 960$



(e) $K = 1920$

Figure 1.6: Maximum achievable native and effective coding rates (information blocklength $K = 480$) (continued from previous page)

2 Code Parameters and Performance Requirements

The goal of this section is to define a common set of parameters and requirements, for the CCSK-NB-code solutions investigated in WP1, combining non-binary (NB) coding and CCSK modulation. The advantage of NB coding can be clearly seen from Section 1.2.2. Moreover, NB coding also provides significant advantages in terms of both demodulation and synchronization, when combined with CCSK modulation [3].

The achievable coding rates results from Section 1.2.1 have highlighted a region of interest, situated above a native coding rate value $R \gtrapprox 1/2$ (Fig. 1.3). Within this region, the maximum achievable effective coding rate R_{eff} is within 1 dB from the capacity of the AWGN channel, for some sufficiently high p value. However, increasing the dimension of the CCSK modulation does also increase the overall complexity of the CCSK-NB-code system. Moreover, the maximum achievable rate in the finite blocklength regime is below the one in the asymptotic case. Thus, in order to allow a flexible trade-off between error correction performance and complexity, we should also consider NB-codes of rate $R < 1/2$. Besides, the design of NB-codes with low coding rates has been identified in the submitted proposal as one of the objectives of the project. Indeed, to maximize the impact of our research, QCSP project focuses on both (i) NB-codes that natively have very low coding rate R, combined with ubiquitous BPSK/QPSK modulations, and (ii) NB codes combined with CCSK modulation to further decrease the effective coding rate.

Accordingly, the following NB-code and CCSK modulation parameters have been agreed on by the QCSP partners, during the project kickoff meeting on October 3rd, 2019.

- Information block size (K, expressed in number of bits)
  - 120, 240, 480, 960, 1920
  - For NB-codes defined over an alphabet of size $q = 2^p$, the above values must be approximated to the nearest multiple of p, if needed (they have been selected such that they are multiple of $p = 6, 8, 10, 12$, except for $K = 120$ which is only a multiple of $p = 6, 10, 12$)
- Native coding rate (R) of the NB-code
  - 1/48, 1/24, 1/12, 1/6, 1/3, 1/2, 2/3
  - Incremental redundancy is optional, but recommended.
- Coded block size (N)
  - Any combination of the above K and R values may be considered, provided that the coded block size $N = K/R$ does not exceed the maximum value $N_{\max} = 24000$ bits.
- Spreading factor (SF) for the CCSK modulation
  - A PN sequence of length $q = 2^p$ may be either shortened (truncated) or extended (by repeating it), to achieve spreading factor values between 1 and $4q/p$.
- Effective coding rate R_{eff} after CCSK modulation
  - Any combination of the above R and SF values may be considered, provided that the effective coding rate $R_{eff} = R/\mathrm{SF}$ is not lower than $1/1000$.
- Channel model
  - AWGN channel, with bipolar ± 1 input

    - 3GPP channel model, to be determined later

- Operating SNR

    - Down to SNR $= -20$ dB, assuming the real noise AWGN channel model. (here, SNR is reported as $-10\log_{10}(\sigma^2)$, where σ^2 is the *real* noise variance)

[Comment] There is a huge number of possible combinations between the NB-code/CCSK parameters we agreed on during the kickoff meting, and it is not clear **how exactly we will compare between the three NB-code families (LDPC, Turbo, Polar)?** Simulating all possible combinations is out of reach, and I'm not even sure this would be really helpful to drawing clear and concise conclusions.

Besides, the above parameters may not be suitable for some of the three NB-code families. For instance, for a NB Polar-code, the mother NB code length is a power of two. Thus, shortening or puncturing should be used just for the sake of adapting to the proposed parameters, although this might not be the most practical solution. For NB-Turbo codes there may also be some constraints on the native coding rate R.

So this is what I propose: Basically, we should only agree on the information block size (K), a minimum value for the effective coding rate (R_{eff}), and a range for the operating SNR. For instance:

- $K = 120, 240, 480, 960, 1920$
- R_{eff} not lower than $1/1000$
- SNR range from -20 to 0 dB.

To compare between the three families of NB-codes:

- Each partner provides **spectral efficiency results**, for each value of K. Here, by spectral efficiency results I mean a plot of the effective coding rate R_{eff} vs. the SNR, where the reported SNR value should be the SNR required to achieve some target FER (say 10^{-4}).
- For each spectral efficiency curve, the distance between successive SNR values should not exceed 1 dB (thus, there should be ≈ 20 (SNR, R_{eff}) points to cover the entire SNR range $[-20, 0]$ dB). We can also divide the SNR range between *mandatory*, (*e.g.*, $[-20, -10]$ dB) and *optional*, ($[-10, 0]$ dB).

Each partner may freely chose the other CCSK/NB-code parameters (non-binary alphabet size, spreading factor, etc.) provided that the above constraints are satisfied. We can then compare the different CCSK/NB-code solutions, by comparing the the spectral efficiency results for each K value. Eventually, we would have a few spectral efficiency curves to compare, and would be easier to draw clear and concise conclusions.

We can then also "annotate" our conclusions by comments on complexity, or detection / synchronization aspects.

By the way, should we add from the beginning some constraints on detection and synchronization? That is, *each partner may freely chose the other CCSK/NB-code parameters, provided that the above constraints are satisfied,* ***and that they allow reliable detection and synchronization*** (in which case we need to define what "reliable detection and synchronization" means, *e.g.*, in terms of probability of missed detection, false alarm, etc.).

Bibliography

[1] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, no. 3-4, pp. 379–423 and 623–656, 1948.

[2] Y. Polyanskiy, H. V. Poor, and S. Verdú, "Channel coding rate in the finite blocklength regime," *IEEE Transactions on Information Theory*, vol. 56, no. 5, pp. 2307–2359, 2010.

[3] O. Abassi, L. Conde-Canencia, M. Mansour, and E. Boutillon, "Non-binary coded ccsk and frequency-domain equalization with simplified llr generation," in *IEEE 24th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*. IEEE, 2013, pp. 1478–1483.